

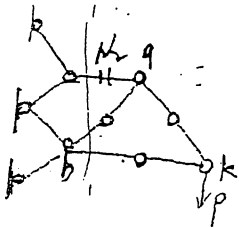
一九八四

解：用灯，IJ，~~GH~~，~~KL~~，~~HM~~ 为零杆

对K结点受力分析

$$\frac{\sqrt{2}}{2}N_1 = N_2 \quad N_1 = \sqrt{2}P \text{ (拉力)}$$

因 ~~IL~~，~~LM~~，~~HM~~，~~EH~~，~~EF~~ 均作为零杆，结构可解



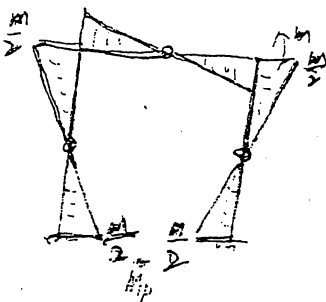
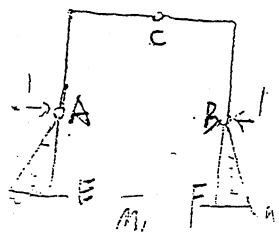
截取H截面左右分析

$$\sum M_b = 0$$

$$\Rightarrow N_2 \cdot 2a = P \cdot 4a$$

$$\Rightarrow N_2 = \frac{1}{2}P$$

解：用 ~~AB~~，~~CD~~，~~EF~~ 为零杆



$$\Delta p = \int \frac{\bar{M}_1 M_p}{EI} ds = \frac{1}{EI} \left(\frac{1}{2} a \times a \cdot \frac{2}{3} \cdot \frac{M}{2} \right) \times 2$$

$$= \frac{-Ma^2}{3EI} \text{ (取反号)} \quad \Delta p = -\frac{1}{EI} \left(\frac{1}{2} \times 80 \times 4 \times \frac{2}{3} \times 2 \right) = -\frac{160}{3EI}$$

$$\Delta t = \frac{t}{2}$$

仅取AC部分沿轴方向t.c.

$$\Delta t = \sum \alpha t \int N ds + \sum \frac{\alpha t}{h} \int \bar{M} ds = 0$$

$$= 0 + \frac{\alpha t}{2h} \cdot \frac{1}{2} a^2 - \frac{\alpha t a^2}{4h}$$

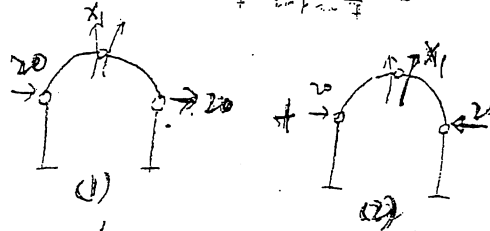
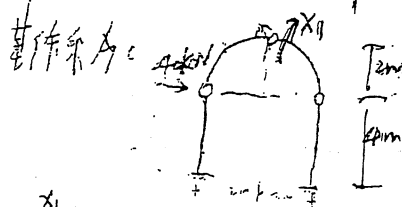
$$\Delta = \frac{\alpha t a^2}{4h} - \frac{Ma^2}{3EI}$$

三杆：尖拱形拱

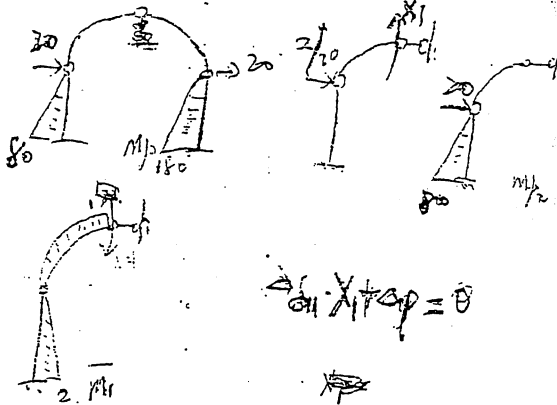
$$W = 3m - 2n - c - c$$

$$m = 3 \quad n = 2 \quad c_0 = 0 \quad c = 6$$

$$W = 9 - 4 - 6 = -1 \text{ 即一次超静定}$$



(1) 取对称轴为式在对称结构中对称轴上的力为0



$$\Delta_{\theta} X_1 + \Delta p = 0$$

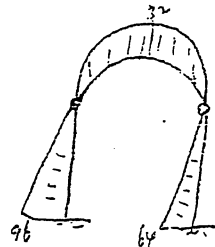
$$M = \bar{M}_1 X_1 + M_{p1} + M_{p2}$$

$$\Delta p = -\frac{1}{EI} \left(\frac{1}{2} \times 80 \times 4 \times \frac{2}{3} \times 2 \right) = -\frac{160}{3EI}$$

$$\Delta_{\theta} = \int \frac{\bar{M}_1^2}{EI} ds = \frac{1}{EI} \int_0^{\frac{\pi}{2}} \left(\frac{1}{2} R \sin \theta \right) \cdot R d\theta +$$

$$= \frac{R^2}{EI} \left(\frac{1}{2} - \frac{1}{2} \right) = -\frac{R^2}{2EI} = -\frac{160}{EI}$$

$$\Rightarrow X_1 = 32$$



天大结构力学竞赛

解：取B、C、E点较水平位移 Δ 为未知量，

$$M_{AB} = -\frac{1}{8}ql^2 - \frac{3EI}{l}\Delta = -20 - 3\frac{EI}{16}\Delta$$

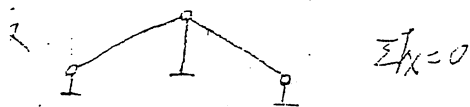
$$M_{BA} = 0, F_{BA} = \frac{3EI}{16}\Delta$$

$$M_{DC} = -\frac{3EI}{6 \times 6}\Delta = -\frac{3EI}{6 \times 6}\Delta = -\frac{EI}{12}\Delta$$

$$F_{CD} = \frac{EI\Delta}{72}$$

$$M_{FE} = -\frac{3EI}{16}\Delta = -\frac{3EI}{16}\Delta$$

$$F_{CE} = \frac{3EI}{64}\Delta$$



$$\sum F_x = 0$$

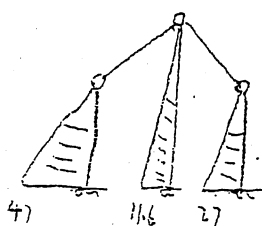
$$\frac{3}{64}EI\Delta - 1 + \frac{1}{72}EI\Delta + \frac{3}{64}EI\Delta = 0$$

$$\Rightarrow EI\Delta = \frac{32 \times 11 \times 9}{31} =$$

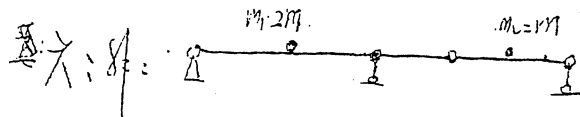
$$M_{AB} = -20 - 3 \times \frac{32 \times 11 \times 9}{31} = -11.6$$

$$M_{DC} = -\frac{1}{12} \times \frac{32 \times 11 \times 9}{31} = -11.6$$

$$M_{FE} = -\frac{3}{16} \times \frac{32 \times 11 \times 9}{31} = -27$$



图



两相邻波为自由振力。

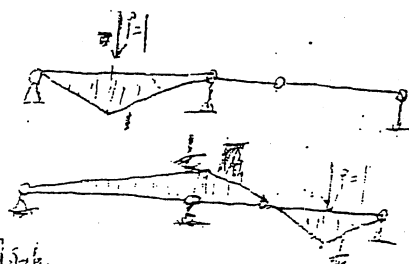
由平衡式

$$(m_1 \delta_{11} - \frac{1}{w_1})Y_1 + m_2 \delta_{12} Y_2 = 0$$

$$m_1 \delta_{21} Y_1 + (m_2 \delta_{22} - \frac{1}{w_2})Y_2 = 0$$

$$\lambda = \frac{1}{w}$$

$$\lambda_{1,2} = \frac{(m_1 \delta_{11} + m_2 \delta_{22}) \pm \sqrt{(m_1 \delta_{11} + m_2 \delta_{22})^2 - 4 m_1 m_2 (\delta_{11} \delta_{22} - \delta_{12} \delta_{21})}}{2}$$



图

图

由图得：

$$\delta_{11} = \frac{1}{EI} \left(\frac{1}{2} \times 2 \times \frac{2}{3} \times \frac{2}{3} \right) = \frac{4}{3EI}$$

$$\delta_{22} = \frac{1}{EI} \left(\frac{1}{2} \times 1 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times 2 + \frac{1}{2} \times 4 \times \frac{1}{2} \times \frac{2}{3} \times \frac{1}{2} \right) = \frac{1}{12EI}$$

$$\delta_{12} = \frac{1}{EI} \left(\frac{1}{2} \times 4 \times \frac{1}{2} \times \frac{1}{3} \right) = \frac{1}{6EI}$$

$$\lambda_{1,2} = \frac{m}{EI} (1.62 \pm 1.214) \Rightarrow \lambda_1 = 2.834 \frac{m}{EI} \Rightarrow w_1 = 0.5 \sqrt{\frac{EI}{m}}$$

$$\lambda_2 = 0.416 \frac{m}{EI} = w_2 = 1.43 \sqrt{\frac{EI}{m}}$$

$$\frac{Y_1}{Y_2} = \frac{-m \delta_{12}}{m_1 \delta_{11} - \frac{1}{w_1}} = \frac{-1}{0.7347}$$

$$\frac{Y_1}{Y_2} = \frac{-m \delta_{12}}{m_2 \delta_{22} - \frac{1}{w_2}} = \frac{1}{4.601}$$



图

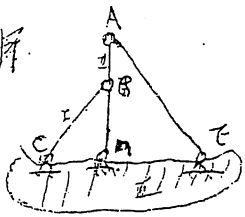


图

九九五

95年

选择

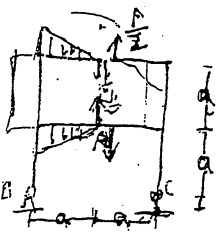


$m=4, n=3, C=6$
 $W=3m-2n-C=12-6-6=0$
 E的约束

取AD杆为刚片II, BC为I, 刚片I为III

I与II交于B, I与III交于C, II与III交于A

ABC不共线 故几何不变 [A]



BC均为水平力
 故上端B, C均发生位移
 截面均发生转动
 故几何不变 [D]

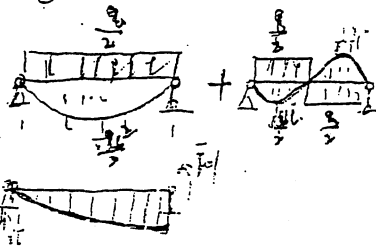
几何不变, 1-8根杆为多余 [C]

[A] $3m-2n-C=0$

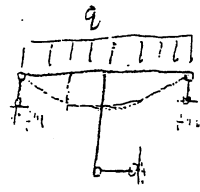
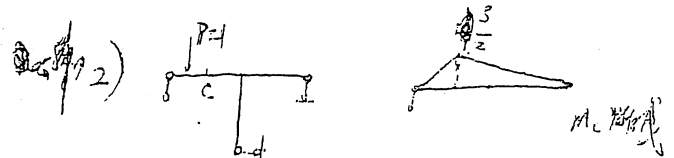
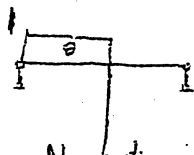
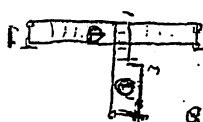
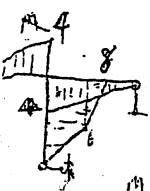
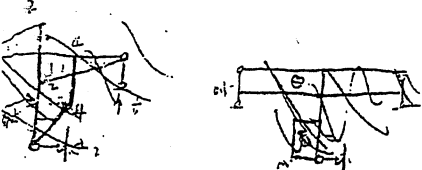
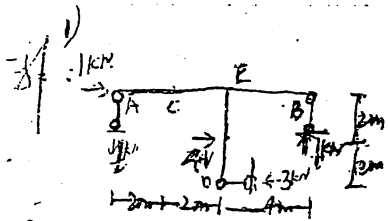
层板结构位移计算

层板结构位移计算

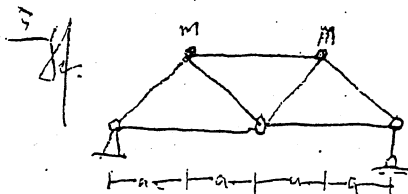
叠加



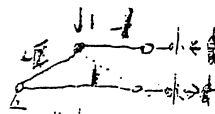
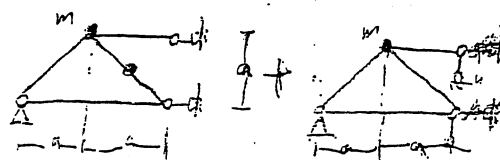
$\Delta = \Delta_1 + \Delta_2 = \frac{1}{EI} \left(\frac{2}{3} \times 4 \times \frac{1}{4} \times \frac{1}{2} \right) + \frac{1}{EI} \left(\frac{2}{3} \times 4 \times \frac{1}{4} \times \frac{1}{2} \right)$
 $= \frac{7963}{48EI} (\uparrow)$



$M_C = \frac{1}{2} \times 8 \times 8 \times 2 - [8 \times 2 \times 1] = 69$

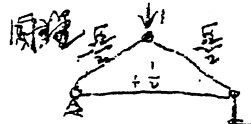


因不计水平运动, 故要前竖向角位移
 因结构对称, 故取半结构



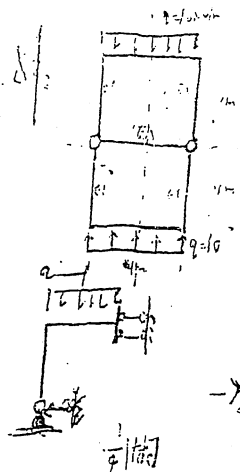
$\delta_1 = \frac{\sum F_i \Delta_i}{EA} = \frac{1}{EA} \left(\frac{1}{2} \times 4 \times \frac{1}{4} \times \frac{1}{2} + \frac{2}{4} \times \frac{1}{4} \times \frac{1}{2} \right)$
 $= \frac{4.49}{EA}$

$W_1 = \sqrt{\frac{I}{m \delta_1}} = \sqrt{\frac{EA}{4.49 m a}}$

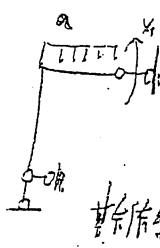


$\delta_2 = \frac{\sum F_i \Delta_i}{EA} = \frac{1}{EA} \left(\frac{1}{2} \times 4 \times \frac{1}{4} \times \frac{1}{2} + \frac{1}{4} \times \frac{1}{4} \times \frac{1}{2} \right)$
 $= \frac{1.9}{EA}$

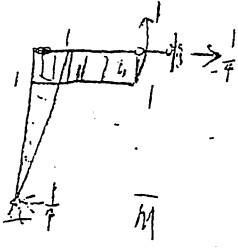
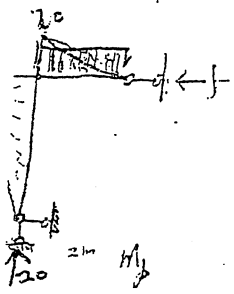
$W_2 = \sqrt{\frac{I}{m \delta_2}} = \sqrt{\frac{EA}{1.9 m a}}$



结构对称，荷载反对称，
可取半结构，并由对称
荷载得

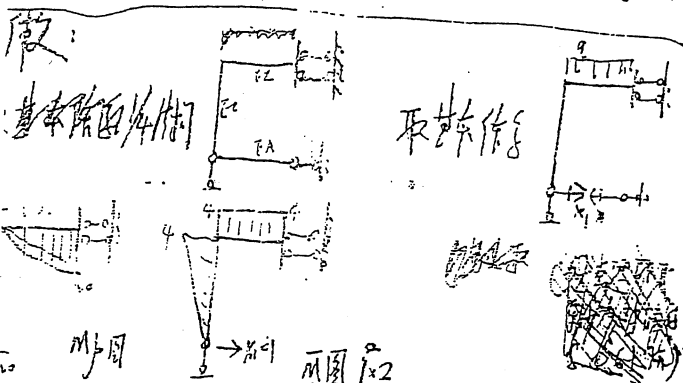


一次超静定

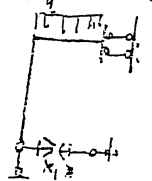


$$\delta_{11} = \frac{1}{EI} \left(\frac{1}{2} \times 4 \times \frac{2}{3} \times 1 + 1 \times 2 \times 1 \right) = \frac{10}{3EI}$$

$$\Delta_{1P} = -\frac{1}{EI} \left(\frac{1}{2} \times 4 \times \frac{2}{3} \times 20 + \frac{1}{3} \times 2 \times 20 \times 1 \right) = -\frac{40}{EI}$$



取半结构

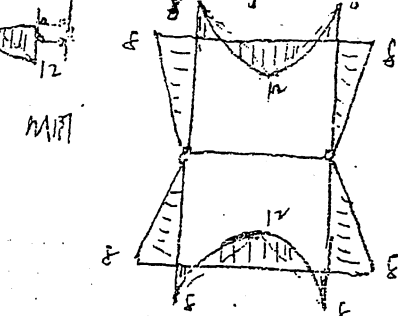


M图

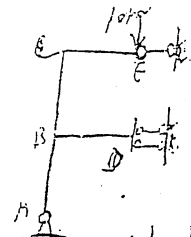
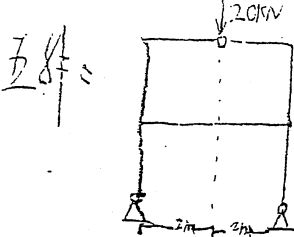
$$\delta_{11} = \frac{1}{EI} \left(\frac{1}{3} \times 4^3 + 4 \times 2 \times 4 \right) = \frac{160}{3EI}$$

$$\Delta_{1P} = -\frac{1}{EI} \left(\frac{2}{3} \times 20 \times 2 \times 4 \right) = -\frac{160 \times 2}{3EI}$$

$$\Delta_{1P} = -\frac{1}{EI} \left(\frac{2}{3} \times 20 \times 2 \times 4 \right) = -\frac{160 \times 2}{3EI}$$



轴力分布



反结构 + 角位移，3个未知数

取半结构，有两个角位移， θ_B, θ_C

$$M_{AB} = 0, M_{BA} = 3i\theta_B, M_{BD} = 2i\theta_B, M_{DB} = -2i\theta_B$$

$$M_{CB} = 2i\theta_B + 4i\theta_C, M_{BC} = -2i\theta_B - 4i\theta_C, M_{CE} = 0, M_{EC} = 0$$

$$\left[\text{取 } i = \frac{EI}{4} \right], \theta_{BC} = \theta_C = 2\theta_B$$

$$\sum M_B = 0: 3i\theta_B + 2i\theta_B + 4i\theta_B + 2i\theta_C = 0$$

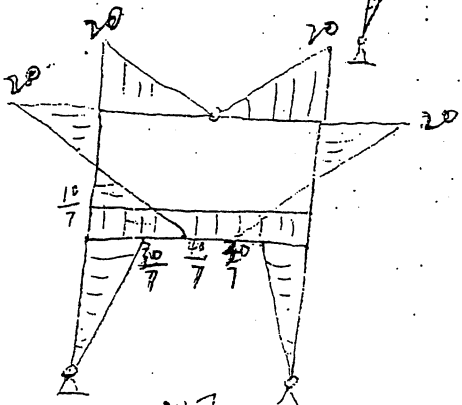
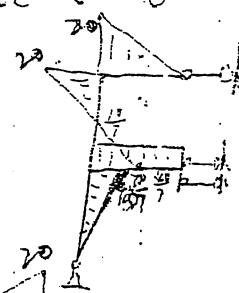
$$\sum M_C = 0: 2i\theta_B + 4i\theta_C - 20 = 0$$

$$\Rightarrow \theta_B = -\frac{10}{7i}, \theta_C = \frac{40}{7i}$$

$$\sum M_B = 3i\theta_B + 2i\theta_B + 4i\theta_B + 2i\theta_C = 0$$

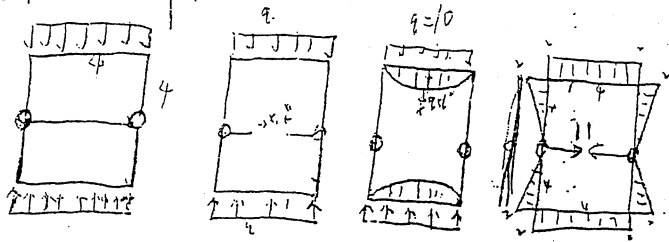
$$\sum M_C = 2i\theta_B + 4i\theta_C - 20 = 0$$

$$\Rightarrow \theta_B = -\frac{10}{7i}, \theta_C = \frac{40}{7i}$$



M图

第四题:

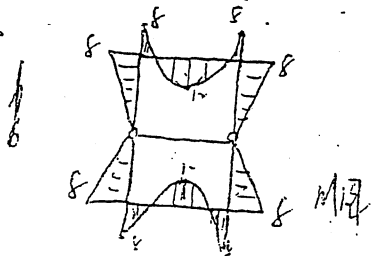


$$\delta_{11} = \frac{1}{EI} \left(2 \times 4 \times 2 + \frac{1}{2} \times 4 \times 2 \times \frac{2}{3} \times 2 \times 4 \right) + \frac{1 \times 4}{EA} = \frac{806}{15EI}$$

$$\delta_{12} = -\frac{1}{EI} \left(\frac{2}{3} \times 4 \times \frac{1}{8} \times 10 \times 4 \times 2 \times 2 \right) = -\frac{640}{3EI}$$

$$X_1 = -\frac{\delta_{12}}{\delta_{11}} = -\frac{\frac{640}{3EI}}{\frac{806}{15EI}} = 4$$

$$M = \bar{M}_1 X_1 + M_p$$

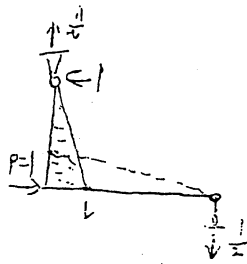
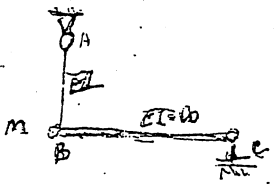
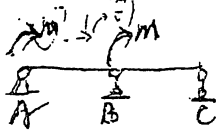


一九九八

1. ✓ 2. ? 3. X



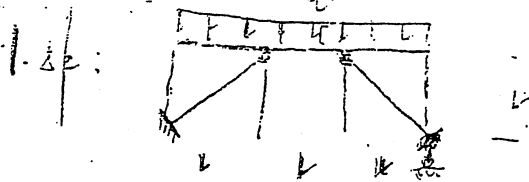
$$\delta_{11} = -\frac{\Delta p}{EI} = -\frac{1}{EI} \left(\frac{1}{2} \times \left(\frac{1}{2} \times m \right) \right) = -\frac{m}{2EI}$$



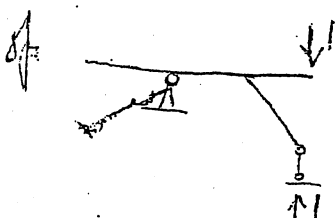
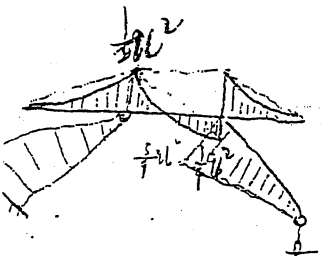
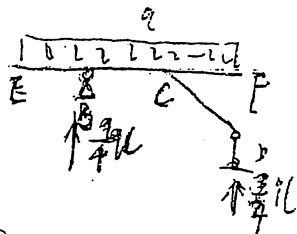
$$\delta_{11} = \frac{1}{EI} \cdot \frac{1}{2} \times L \times \left(\frac{1}{2} \times L \right) = \frac{L^3}{2EI}$$

$$W = \sqrt{\frac{1}{m \delta_{11}}} = \sqrt{\frac{2EI}{3mL^3}}$$

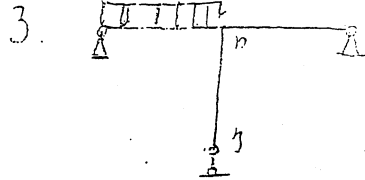
[A]



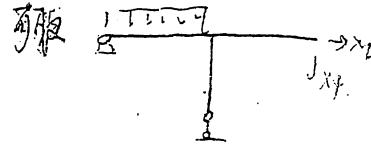
刚架前附属结构



$$\Delta F = -F_2 \cdot C = -(-1) \cdot 0.01 = 0.01 \text{ m}$$

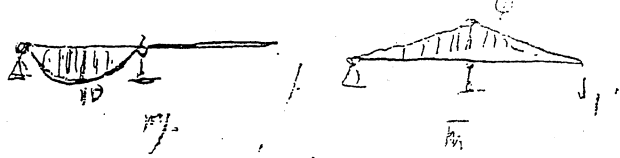
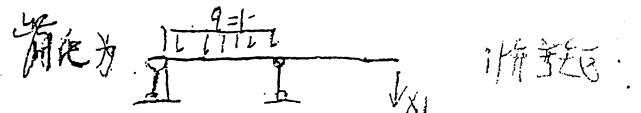


结构为两次超静定



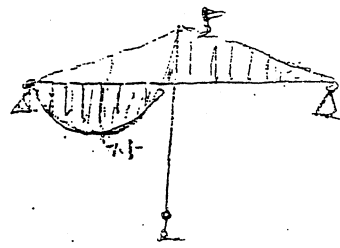
因 \$X_2\$ 为 0, 则杆件上水平作用力为 0

则中点 B 点杆件所受剪力为 0 故结构为一次超静定

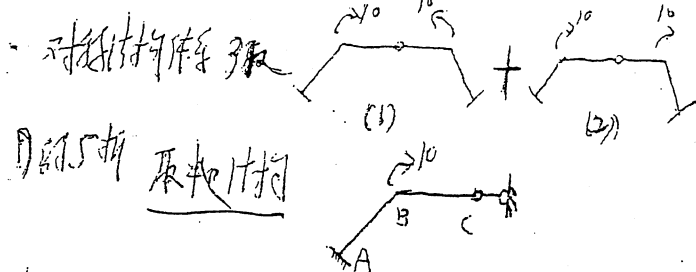
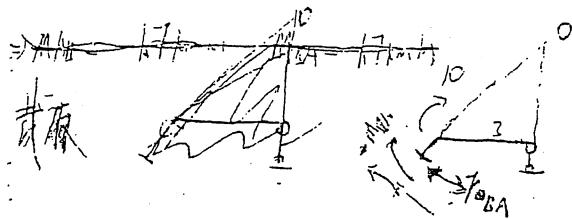
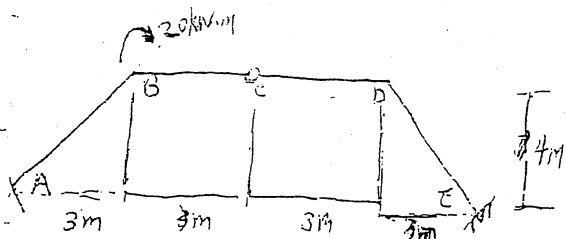


$$X_1 = -\frac{\Delta p}{\delta_{11}} = -\frac{\frac{1}{EI} \cdot \left(\frac{1}{2} \times \left(\frac{1}{2} \times m \right) \right)}{\frac{1}{EI} \cdot \left(\frac{1}{2} \times \left(\frac{1}{2} \times m \right) \right)} = \frac{5}{4}$$

$$M = M_1 X_1 + M_2$$



正确



$$\sum M_i = 0 \quad F_{QBA} \cdot 5 + 10 + M_{BA} = 0$$

$$F_{QBA} = \frac{M_{AB} - M_{BA}}{5} \cdot 5 + 10 + M_{BA} = 0$$

$$M_{AB} - 2M_{BA} + 10 = 0 \quad \text{①}$$

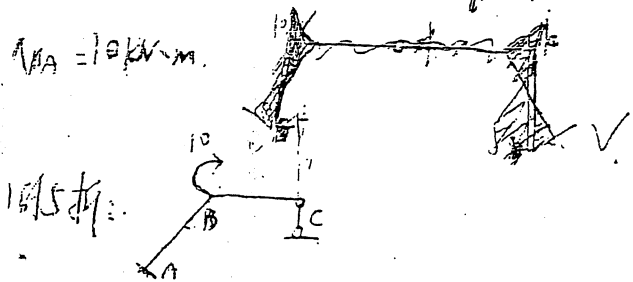
转角为 θ_B

$$\sum \theta_i = 0 \quad \theta_{BA} = ?$$

$$M_{AB} = 2EI\theta_B = 4 \cdot \frac{2EI}{5} \theta_B \quad M_{BA} = 4i\theta_B = \frac{4EI}{5} \theta_B$$

$$M_{BC} = M_{CB} = 0 \quad \sum M_i = 0 \quad \frac{4EI}{5} \theta_B = 10$$

$$\Rightarrow EI\theta_B = \frac{25}{2} \quad M_{AB} = 5 \text{ kN·m}$$



转角为 θ_B, Δ_{BC}

$$M_{AB} = 2EI\theta_B - \frac{6i}{L} \cdot \frac{1}{4} \Delta_{BC} \quad F_{QBA} = \frac{2EI}{5} \theta_B - \frac{3EI}{20} \Delta_{BC}$$

$$M_{BA} = 4i\theta_B - \frac{6i}{L} \cdot \frac{1}{4} \Delta_{BC} \quad F_{QBC} = \frac{4EI}{5} \theta_B - \frac{3EI}{20} \Delta_{BC}$$

$$M_{BC} = \frac{2EI}{L} \Delta_{BC} \cdot \frac{3}{4} \Delta_{BC} = \frac{EI}{2} \Delta_{BC} + EI\theta_B$$

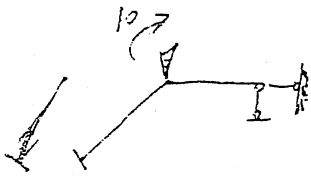
$$M_{BC} = 10 \Rightarrow \frac{EI}{2} \Delta_{BC} + EI\theta_B = 10$$

$$\sum F_x = 0$$

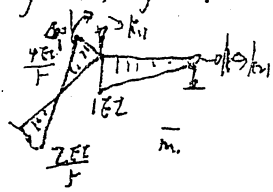
$$F_{QBA} = 0 \quad F_{QBC} = 0$$

$$\frac{6EI}{5} \theta_B - \frac{3EI}{20} \Delta_{BC} = 0 \quad \frac{EI}{2} \Delta_{BC} = 10$$

2) 法并 (2)
74 6 15 8 16
12.11

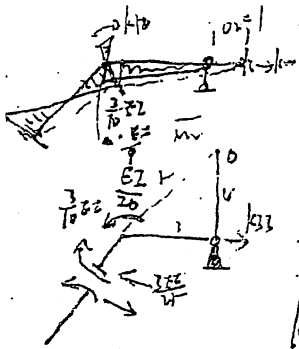


$$F_{11} = 1, F_{12} = 0$$



$$k_{11} = \frac{4}{l} EI + 1 \cdot EI = \frac{5}{l} EI$$

k_{12} 由 k_{21} 并



$$k_{12} = -\frac{3}{10} EI + \frac{EI}{4} = -\frac{1}{10} EI$$

$$k_{21} = 0$$

$$k_{22} = \frac{3}{4} EI + \frac{3}{10} EI$$

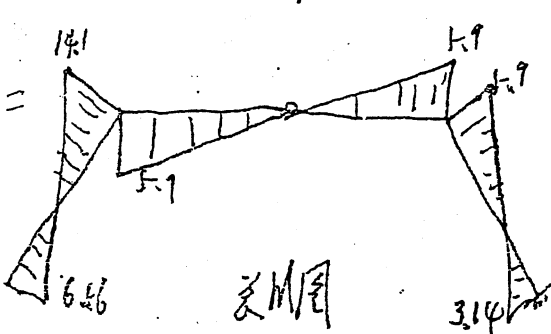
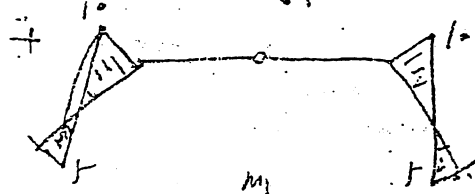
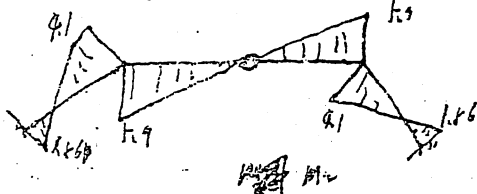
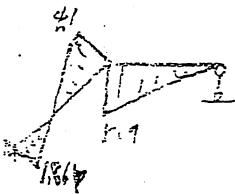
$$k_{33} = \frac{9}{40} EI$$

$$k_{11} \delta_1 + k_{12} \delta_2 + F_{11} = 0$$

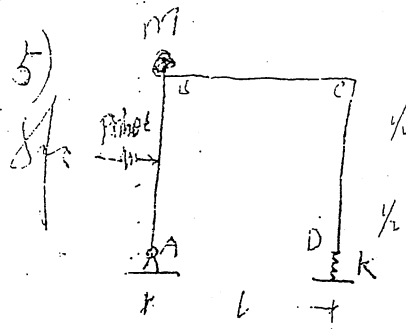
$$k_{21} \delta_1 + k_{22} \delta_2 + F_{12} = 0$$

$$\Rightarrow \begin{cases} \delta_1 = 5.67 \\ \delta_2 = 1.24 \end{cases}$$

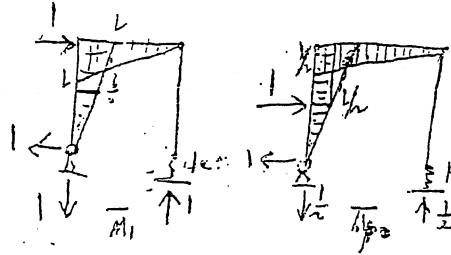
$$M_1 = k_{11} \delta_1 + k_{12} \delta_2$$



交用图



单自设



$$\delta_{11} = \frac{1}{EI} \left(\frac{1}{2} \times l \times \frac{1}{2} \times l \times 2 \right) = -\frac{1}{2} EI$$

$$= \frac{2l^3}{3EI} + 1 \cdot \frac{1}{k} = \frac{2l^3}{3EI} + \frac{2l^3}{EI} = \frac{8l^3}{3EI}$$

$$\delta_{12} = \frac{1}{EI} \left(\frac{1}{2} \times l \times \frac{1}{2} \times \frac{2}{3} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{2}{3} \times \frac{1}{2} \right. \\ \left. + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{2}{3} \times \frac{1}{2} \right) = -\frac{1}{2} EI$$

$$= \frac{19l^3}{98EI} + \frac{1}{2} \cdot \frac{1}{k} = \frac{67l^3}{48EI}$$

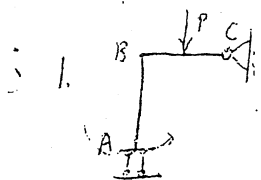
$$W = \sqrt{\frac{1}{m \delta_{11}}} = \sqrt{\frac{3EI}{8m l^3}}$$

$$\beta = \frac{1}{1 - \frac{W}{m}} = \frac{1}{1 - \frac{1}{4}} = \frac{4}{3}$$

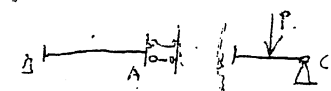
$$M_{max} = \beta \cdot P \cdot \delta_{11} = \frac{4}{3} \times P \times \frac{67l^3}{48EI}$$

$$= \frac{67Pl^3}{36EI}$$

七、八



用矩阵位移法. θ_B 为未知数



$$M_{BA} = i \theta_B, M_{AB} = -i \theta_B$$

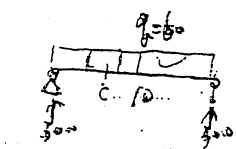
$$M_{BC} = 5i \theta_B - \frac{3}{16} P L \quad \text{由 } \sum M_i = 0 \Rightarrow \theta_B = \frac{3PL}{64i}$$

$$M_{AB} = -\frac{3}{64} PL \quad \text{左端受拉}$$

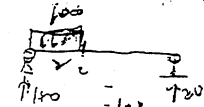
$$M = \frac{1}{1 - \left(\frac{v}{w}\right)^2}$$

$$0 \leq \frac{v}{w} \leq 1 \Rightarrow 0 \leq \left(\frac{v}{w}\right)^2 \leq 1$$

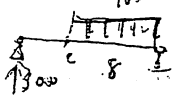
$$0 \leq 1 - \left(\frac{v}{w}\right)^2 \leq 1 \Rightarrow M \in [1, +\infty)$$



$$V_0 = 180 \downarrow$$



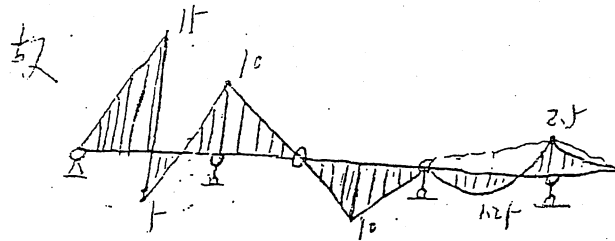
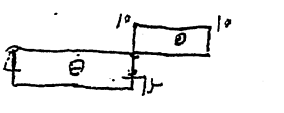
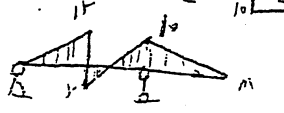
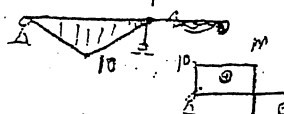
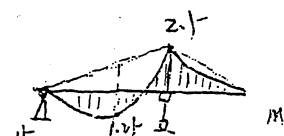
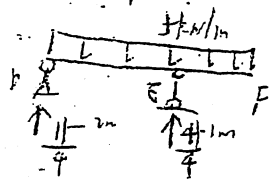
$$V_0' = 20 \uparrow \quad \therefore V_{max} = 60 \downarrow$$



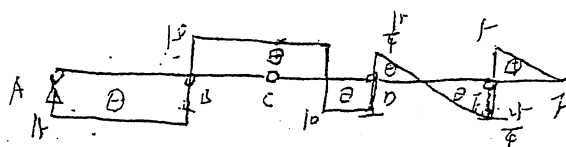
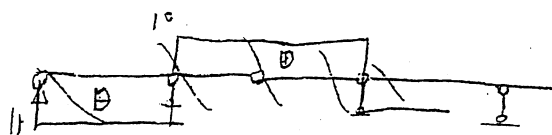
$$V_0' = 50 \downarrow \quad V_{max} = 100 \downarrow$$

1. [C] 2. 3. D.

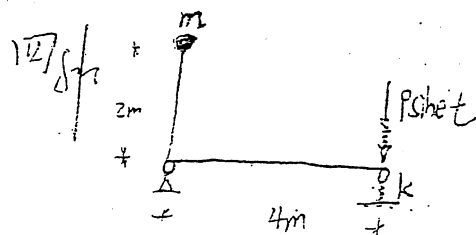
静定与超静定, 依此采取附属结构计算。



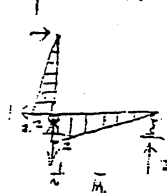
M图



Q图



静定/支

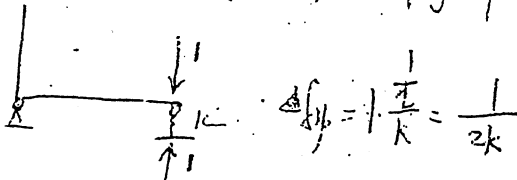


$$\Delta H = \frac{1}{E} \left(\frac{1}{2} \times 2 \times \frac{1}{3} \times 2 + \frac{1}{2} \times 4 \times 2 \times \frac{1}{3} \times 2 \right) + \frac{1}{2} \times \frac{P}{K}$$

$$= \frac{8}{E} + \frac{1}{4K} = \frac{4EZK}{4EZK}$$

$$W = \sqrt{\frac{1}{m \delta_1}} = \sqrt{\frac{4EZK}{m(32K + EZ)}}$$

$$\beta = \frac{1}{1 - \frac{v}{w}}$$



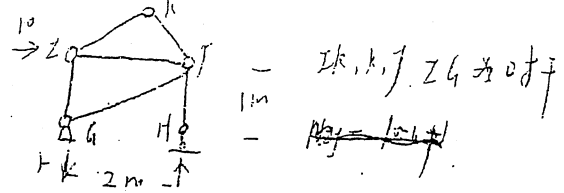
$$\Delta \delta_p = 1 \cdot \frac{1}{K} = \frac{1}{2K}$$

$$y_{max} = \beta \cdot y_{st} = \beta \cdot P \cdot \delta_p = \frac{4EZK}{4EZK - m \omega^2 (32K + EZ)} \cdot P \cdot \frac{1}{2K}$$

$$= \frac{2EZP}{4EZK - m \omega^2 (32K + EZ)}$$

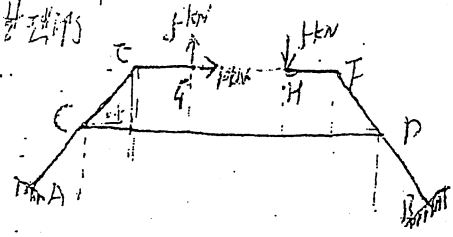
★ 无限刚性杆

① 发生折角后

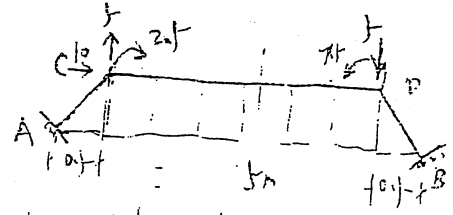


$R_H = F \sin(\theta)$ $R_V = F \cos(\theta)$ $V_G = 1/2 \cdot 1 \cdot 1$

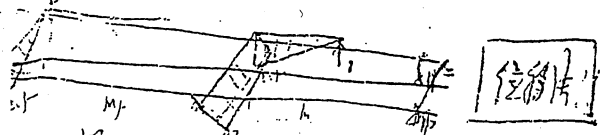
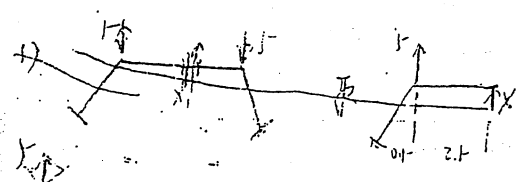
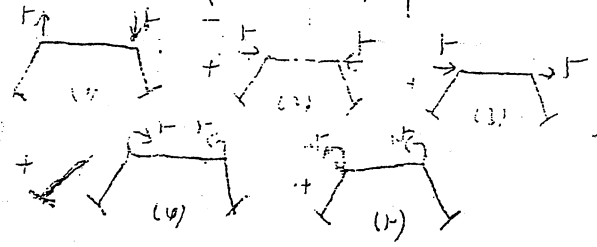
② 取杆件



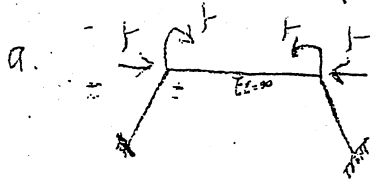
C G 与 D H 为无限刚性杆，内力分析
 故取 5 折 A C D B 起始与材料内力状态



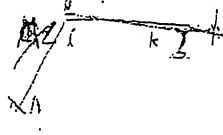
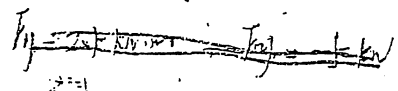
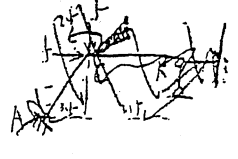
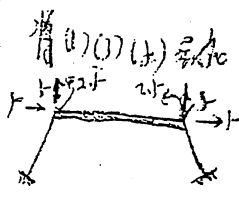
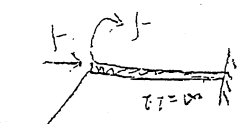
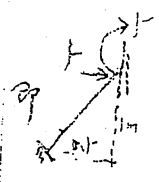
取杆件 1 与 2 组合分析



将杆件 1 与 2 组合

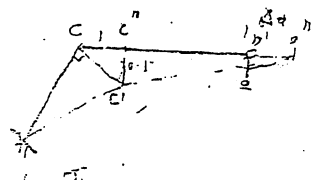
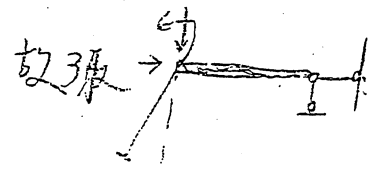


取杆件 1 与 2



取

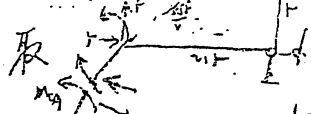
实际上 D 处水平位移与 C 处转角
 有必然联系 至于 1111 无限刚性
 杆 1 与 2 杆上各点



$\theta_c \approx \theta_g = \frac{CC''}{C'D''} = \frac{CC''}{C'D'} = \frac{\Delta}{2L} = \frac{\Delta}{L} (T)$

$M_{CA} = -4i\theta_c - \frac{6i}{L}\Delta = -\frac{4i}{L}\Delta - \frac{6i}{L}\Delta = -\frac{10i}{L}\Delta$

$M_{AC} = -2i\theta_c - \frac{6i}{L}\Delta = -\frac{4i}{L}\Delta - \frac{6i}{L}\Delta = -\frac{10i}{L}\Delta$

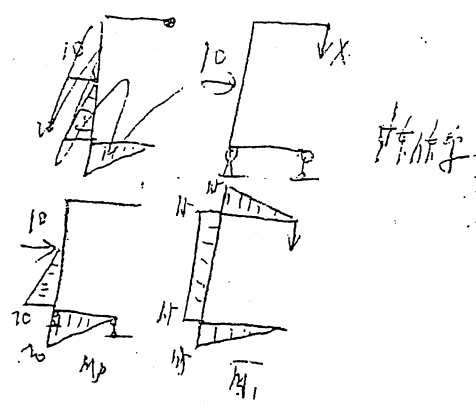
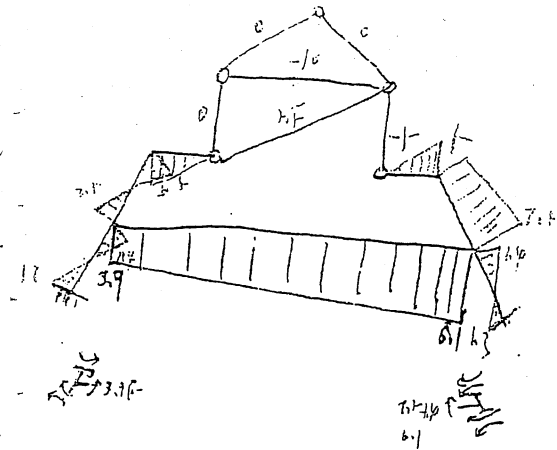


$\sum M_O = 0$

$|Q_A| \frac{L}{2} + |M_A| = 2L + L = 3L$

$\frac{24EZ}{L^3} \Delta \cdot \frac{L}{2} + \frac{6EZ}{L^2} \Delta = 3L$

$\Rightarrow \Delta = \frac{16FL}{72EZ} \Rightarrow M_{CA} = -1.9$
 $M_{AC} = -1.3$

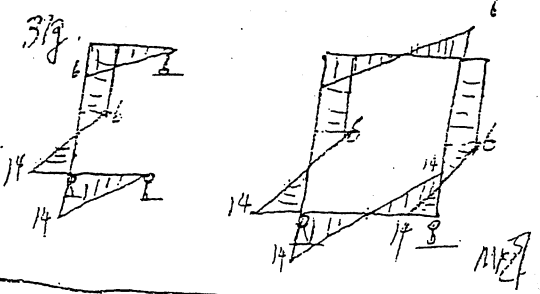


$$\Delta f_1 = \frac{1}{EI} \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times 2 + \frac{1}{2} \times 4 \times \frac{1}{2} \right) = \frac{41}{482}$$

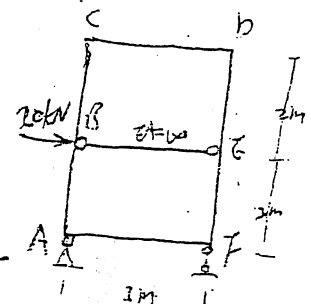
$$\Delta f_2 = \frac{1}{EI} \left(\frac{1}{2} \times 20 \times 2 \times \frac{1}{2} + \frac{1}{2} \times 20 \times \frac{1}{2} \times \frac{2}{3} \times \frac{1}{2} \right) = \frac{41}{482}$$

$$\therefore X = -\frac{\Delta f}{\delta_{11}} = -4$$

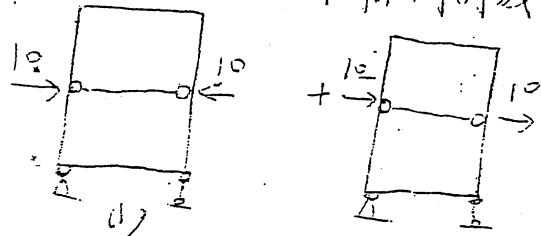
$$\therefore M = M_1 X + M_2$$



12 kN

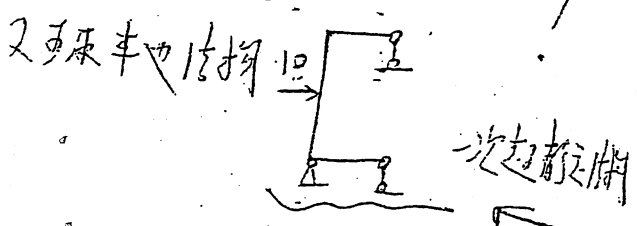
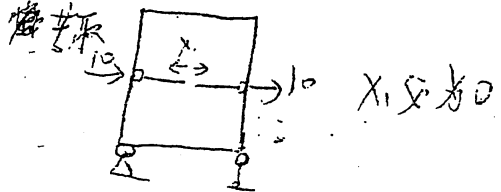


由题意可将原结构拆成(1)(2)

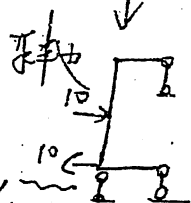
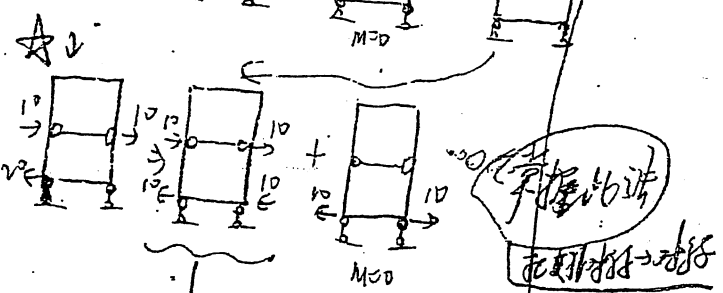
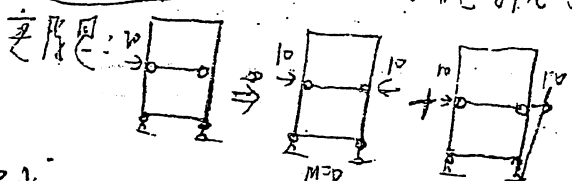


1) 因为轴力杆 BE, EA=0, 故其内力为 0, $N_{BE} = -10 \text{ kN}$ (压力)

2) 取对称结构反对称荷载作用下

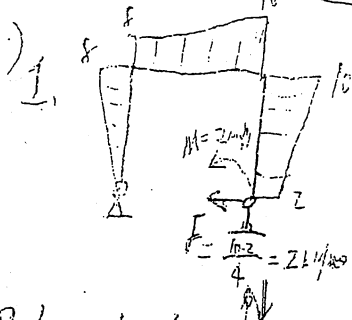


本题解法3: 把对称结构拆成对称和反对称

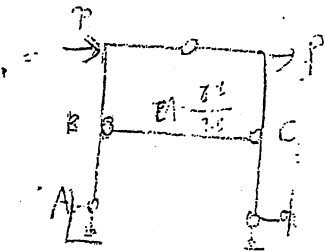


对称结构拆成!! 反对称结构拆成!! 反对称!!

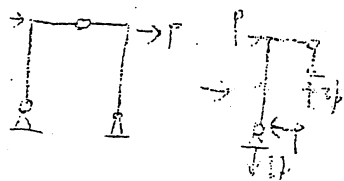
一九九一



在E处有顺时针方向的力矩为2kN.m的力
E处水平向左的荷载2kN的作用。

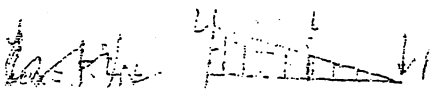
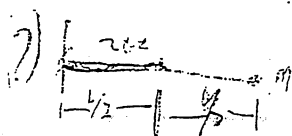


对E处取矩，对E处取矩， $N_{BC} = 0$



$$H_A = P (\leftarrow)$$

$$V_A = 2P (\downarrow)$$



$$\delta_{11} = \frac{1}{EI} \cdot \frac{1}{3} \left(\frac{l}{2} \right)^3 + \frac{1}{2EI} \cdot \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{l}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{l}{2} \right)$$

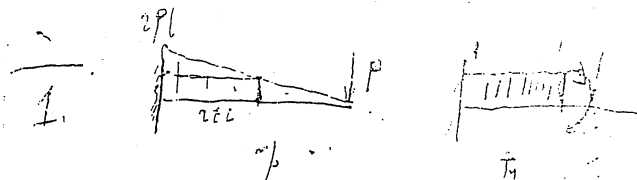
$$= \frac{3l^3}{16EI}$$

$$W = \sqrt{\frac{1}{m \delta_{11}}} = \sqrt{\frac{16EI}{3ml^3}}$$

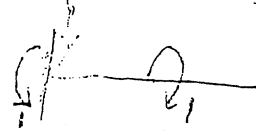
$$\text{位移 } \Delta = \sqrt{y_0^2 + \frac{v_0^2}{w}}$$

$$y_0 = \sqrt{y_0^2 + \frac{v_0^2}{w}} \Rightarrow v_0 = \sqrt{\frac{2EI}{ml^3}}$$

$$= \sqrt{y_0^2 + \frac{v_0^2}{w}} \quad (\text{位移 } \Delta \text{ 为反力!!!})$$



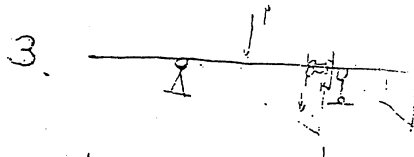
$$\delta_1 = \frac{1}{2EI} (1 \times l \cdot \frac{1}{2} \times \frac{1}{2}) = \frac{3l^2}{4EI} (2)$$



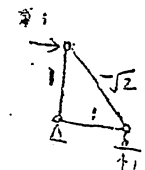
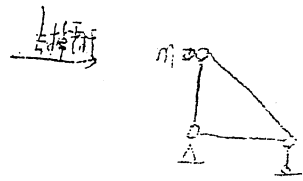
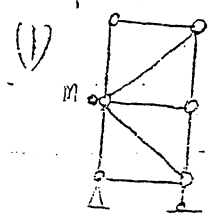
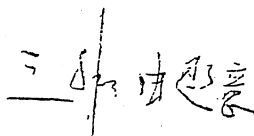
$$\Delta_2 = -\sum F_i \cdot \Delta_i = -(-1) \cdot \varphi = \varphi (1)$$

$$\Delta = 0, \text{ 则 } \varphi = \frac{3Pl^2}{4EI} + \varphi = [B]$$

2. (?)



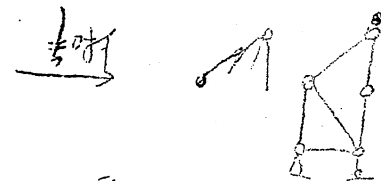
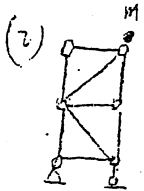
[B]



$$\delta_{11} = \frac{\sum F_i^2 l}{EA} = \frac{1}{EA} (1^4 + 1^4 + 2 \times 4^2)$$

$$= \frac{18}{EA}$$

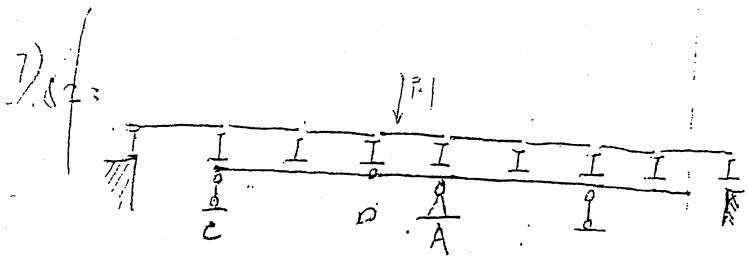
$$W_1 = \sqrt{\frac{1}{m \delta_{11}}} = \sqrt{\frac{EA}{m(18+16)}}$$



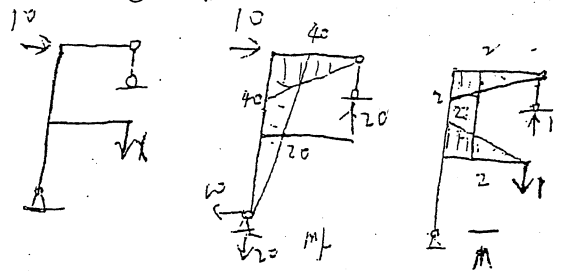
$$\delta_{11} = \frac{\sum F_i^2 l}{EA} = \frac{1}{EA} (1^4 + 1^4 + 2 \times 4^2 + 2 \times 4^2)$$

$$= \frac{28+16}{EA}$$

$$W_1 = \sqrt{\frac{1}{m \delta_{11}}} = \sqrt{\frac{EA}{(28+16)m}}$$



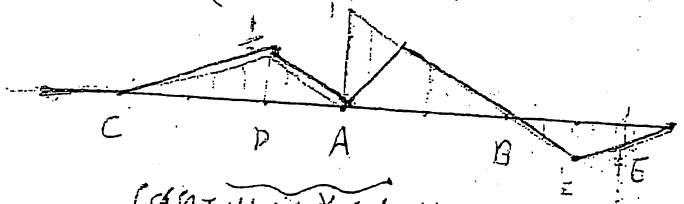
取半跨材料



类似心板如下片



此题位置可视为直线段



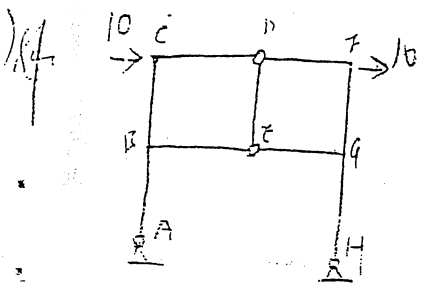
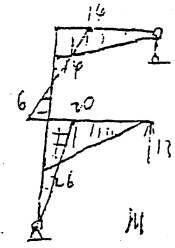
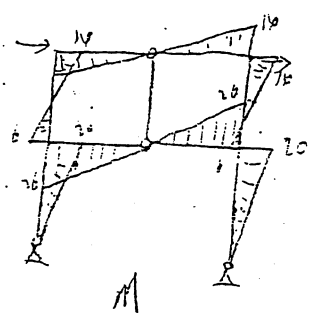
(任何两片均为直线!!!)

$$\delta_{11} = \frac{1}{EI} \left(\frac{1}{2} \times 2 \times 2 \times \frac{2}{3} \times 2 + 2 \times 2 \times 2 \right) = \frac{40}{3EI}$$

$$\Delta p = \frac{1}{EI} \left(\frac{1}{2} \times 2 \times 2 \times \frac{2}{3} \times 40 + 2 \times 2 \times \frac{20+40}{2} \right) = \frac{120}{3EI}$$

$$X_1 = - \frac{\Delta p}{\delta_{11}} = -13$$

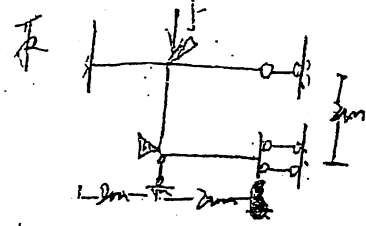
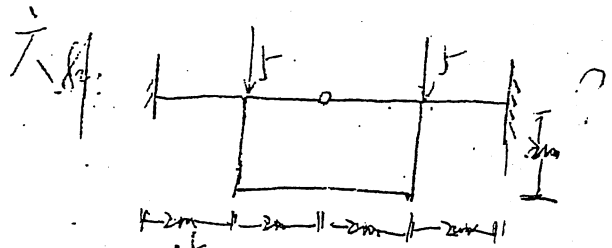
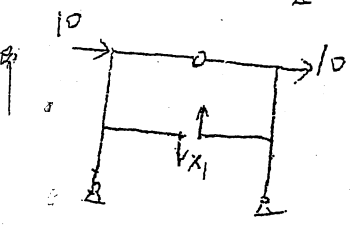
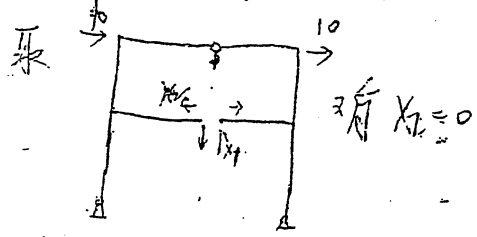
$$M = \overline{M}_1 \cdot X_1 + M_p$$



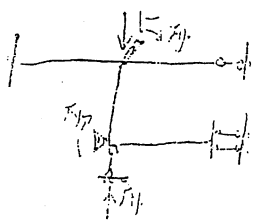
取 F17 $W = 3M - 2n - 6 - C$
 $= 3 \cdot 3 - 2 \cdot 4 - 0 - 4 = -3$

对称结构反对称荷载作用下

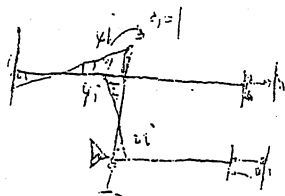
$N_{DE} = 0$



$$\begin{cases} k_{11}\Delta_1 + k_{12}\Delta_2 + k_{13}\Delta_3 + F_{1p} = 0 \\ k_{21}\Delta_1 + k_{22}\Delta_2 + k_{23}\Delta_3 + F_{2p} = 0 \\ k_{31}\Delta_1 + k_{32}\Delta_2 + k_{33}\Delta_3 + F_{3p} = 0 \end{cases}$$



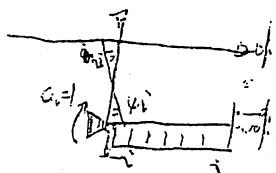
$$F_{j0} = -1, F_{j1} = F_{j2} = 0$$



$$F_{j1} = 8i$$

$$F_{j2} = 2i$$

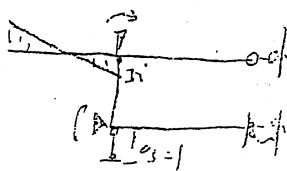
$$F_{j3} = -3i$$



$$F_{j1} = 2i$$

$$F_{j2} = 1i$$

$$F_{j3} = 0$$



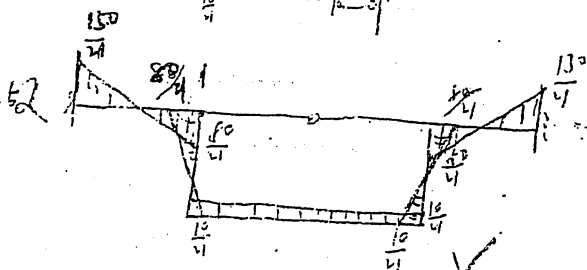
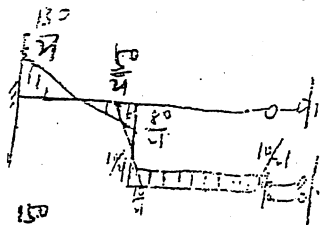
$$F_{j1} = -3i$$

$$F_{j2} = 0$$

$$F_{j3} = 3i$$

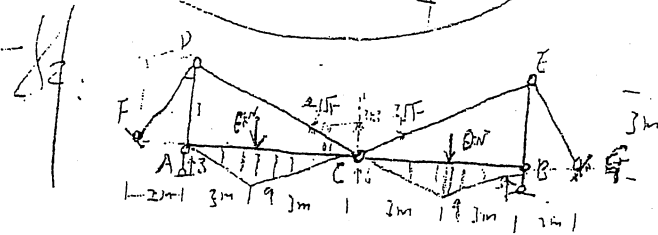
$$\begin{cases} 8i\alpha_1 + 2i\alpha_2 - 7i\alpha_3 = 0 \\ 2i\alpha_1 + 1i\alpha_2 = 0 \\ -N\alpha_1 + 3i\alpha_3 - 1 = 0 \end{cases} \Rightarrow \begin{cases} \alpha_1 = \frac{2}{11} \\ \alpha_2 = -\frac{10}{21} \\ \alpha_3 = \frac{70}{71} \end{cases}$$

$$M = M_1\alpha_1 + M_2\alpha_2 + M_3\alpha_3$$



M图

- 2.2.4



1. 结构受力图. 只折半也半折

$V_1 = 1$. 取C点. $\sum F_y = 0$.

$$N_{CD} = N_{CE} = 3\sqrt{5} \text{ (拉)}$$

取D点平衡. $\sum F_x = 0$.

$$N_{DF} \cdot \frac{2}{\sqrt{5}} = N_{DE} \cdot \frac{2}{\sqrt{5}} = 6$$

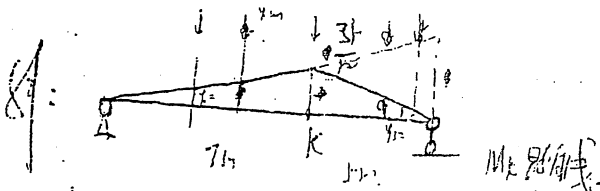
$$N_{DF} = 3\sqrt{3} \text{ (拉)} \quad \sum F_y = 0 \quad N_{AD} = N_{DE} = -12 \text{ kN (压)}$$

1. 结构各数据, 二力杆:

$$N_{CD} = N_{CE} = 3\sqrt{5} \text{ (拉)} \quad N_{AD} = N_{DE} = -12 \text{ kN (压)}$$

$$N_{DF} = N_{EF} = 3\sqrt{3} \text{ (拉)}$$

2. 杆件受力图同前. ✓

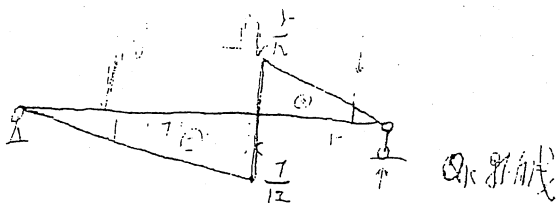


将荷载等效为集中力, 令中间荷载处于2/3处.

$$y_1 = \frac{1}{4}, \quad y_3 = \frac{7}{12}$$

$$M_{max} = P_1 \cdot y_1 + P_2 \cdot y_2 + P_3 \cdot y_3 = 1 \times \frac{1}{4} + 1 \times \frac{1}{2} + 1 \times \frac{7}{12} = \frac{91}{4} \text{ kNm}$$

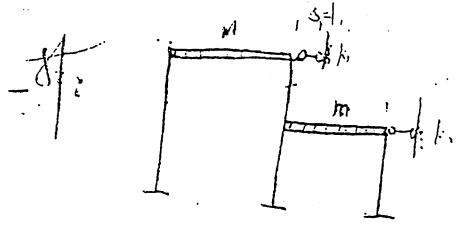
(2. 假定荷载为集中力, 同左向右依次取 B, P, P)



✓ P_1 及 k_{11}

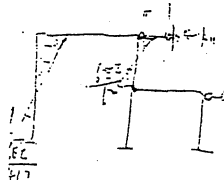
$$V_{max} = 5 \times \frac{1}{12} + 5 \times \frac{1}{12} \times \frac{1}{5} = \frac{1}{2}$$

$$V_{min} = - \left(5 \times \frac{7}{12} + 5 \times \frac{7}{12} \times \frac{3}{7} \right) = -\frac{25}{8} \checkmark$$



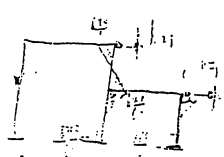
由这8个自由度求得平衡方程

$$\Delta_1 = 1 \text{ 吋}$$



$$k_{11} = \frac{7EI}{21^3} + \frac{12EI}{1^3} = \frac{27EI}{21^2}$$

$$k_{12} = -\frac{12EI}{1^2} = k_{21}$$



$$k_{22} = \frac{12EI}{3^3} \times 3 = \frac{36EI}{1^3}$$

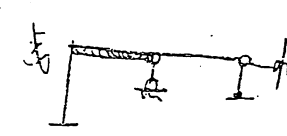
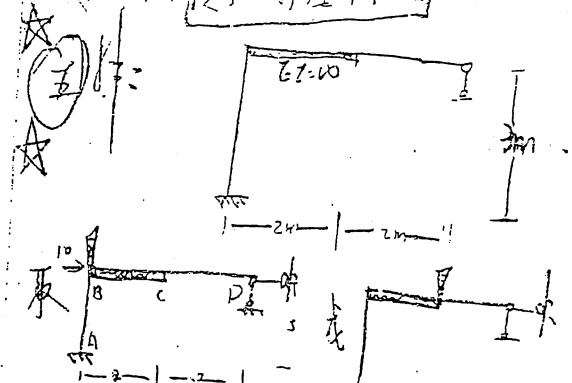
$$\begin{vmatrix} k_{11} - m_1 \omega^2 & k_{12} \\ k_{21} & k_{22} - m_2 \omega^2 \end{vmatrix} = 0$$

$$\Rightarrow W_1^2 = \frac{1}{2} \left(\frac{k_{11}}{m_1} + \frac{k_{22}}{m_2} \right) \pm \sqrt{\left[\frac{1}{2} \left(\frac{k_{11}}{m_1} + \frac{k_{22}}{m_2} \right) \right]^2 - \frac{k_{12}^2}{m_1 m_2}}$$

$$\Rightarrow W_1 = 2.881 \sqrt{\frac{EI}{m_1 l^3}}$$

$$W_2 = 6.4186 \sqrt{\frac{EI}{m_1 l^3}} \checkmark$$

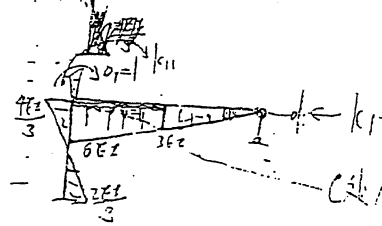
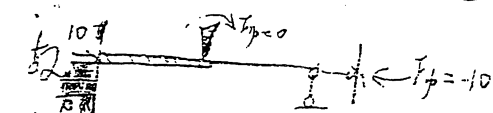
四) 材料 / 无限刚性杆



Δ_{BBD}
三杆材料刚度相等
因为三杆长度相等



$$k_{12} = \theta_c \quad \Delta c = \theta_c l_{sc}$$



$$\frac{12EI}{2^3} - \frac{12EI}{2 \times 2} \left(\frac{1}{2} \right) = 36EI$$

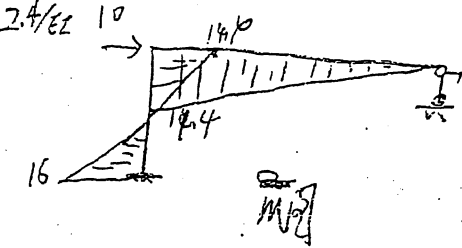
$$k_{11} = \left(6 + \frac{4}{3} \right) EI, \quad k_{12} = -\frac{2}{3} EI = k_{21}$$

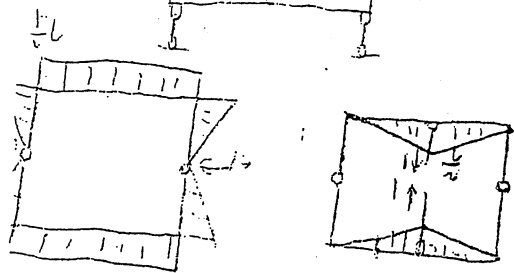
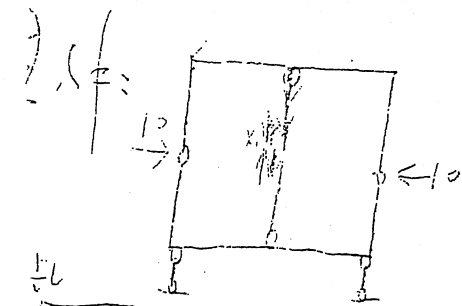
$$k_{22} = \frac{4}{9} EI$$

$$\begin{cases} k_{11} \Delta_1 + k_{12} \Delta_2 + F_p = 0 \\ k_{21} \Delta_1 + k_{22} \Delta_2 + F_p = 0 \end{cases} \Rightarrow \begin{cases} \frac{22}{3} \Delta_1 - \frac{2}{3} \Delta_2 = 0 \\ -\frac{2}{3} \Delta_1 + \frac{4}{9} \Delta_2 = \frac{10}{EI} \end{cases}$$

$$\Delta_1 = \frac{41}{19 EI} = 2.4/EI \quad 10$$

$$\Delta_2 = 26.4/EI$$





$$y_{max} = \beta y_{ic} = \beta p \delta_{12}$$

$$\beta = \frac{1}{1 - \frac{e^2}{w}} = \frac{3EI}{3EI - 4m\tilde{e}L^3}$$

$$y_{max} = \frac{3 p L^3}{48EI - 64m\tilde{e}L^3}$$



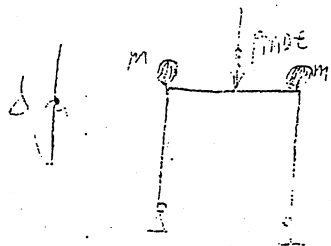
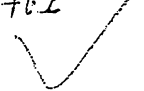
轴力对截面作拉力时，也有位移量，另

$$\delta_{11} = \frac{1}{EI} \cdot \frac{1}{2} \times \frac{L}{2} \times \frac{L}{2} \times \frac{L}{2} \times \frac{L}{2} \times 2 = \frac{L^3}{16EI}$$

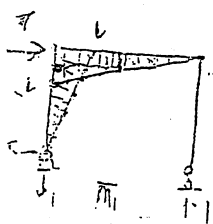
$$\Delta p = -\frac{1}{EI} \cdot \frac{1}{2} \times 2 \times \frac{L}{2} \times \frac{L}{2} \times 2 = -\frac{L^2}{2EI}$$

$$X_1 = \frac{-\Delta p}{\delta_{11}} = \frac{1}{2} = 0.5$$

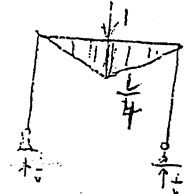
$$X_1 = -\frac{\Delta p}{\delta_{11}} = \frac{1 \cdot A \cdot L^2}{2A L^2 + 6EI}$$



$$y(t) = \sum \ddot{y}_i(t) \delta_{11} + \ddot{p}(t) \delta_{12}$$



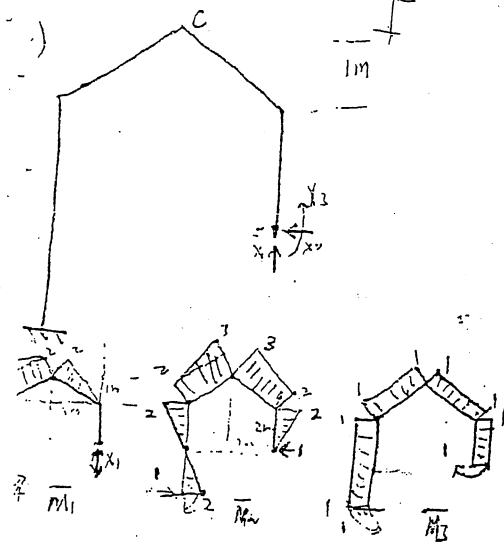
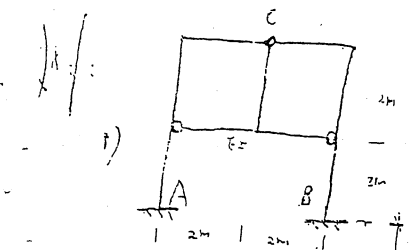
$$\delta_{11} = \frac{2L^3}{16EI}$$



$$\delta_{12} = \frac{L^3}{16EI}$$

$$w = \sqrt{\frac{1}{\sum \delta_{11}}} = \sqrt{\frac{3EI}{4mL^3}}$$

支座的位移
 位移互等定理
 (?)



$$t_0 = \frac{10-0}{2} = 0 \quad \Delta t = 10 - (t_0) = 20$$

$$\Delta t = 2 \cdot t_0 \int \sqrt{s} ds + \frac{\Delta t}{h} \int \sqrt{s} ds$$

$$t = -\frac{2 \cdot 20}{h} \cdot (4 \times 4 + \frac{4+2}{2} \times 2 \sqrt{2} + \frac{1}{2} \times 2 \times \sqrt{2}) =$$

$$t = -\frac{2 \cdot 20}{h} (\frac{1}{2} \times 2 \times 2 + \frac{3+2}{2} \times 2 \sqrt{2} \times 2 + 0) =$$

$$t = -\frac{2 \cdot 20}{h} [1 \times (2 + \sqrt{2} \times 2 + 4)] =$$

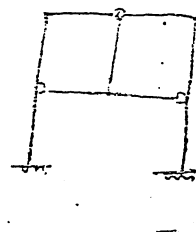
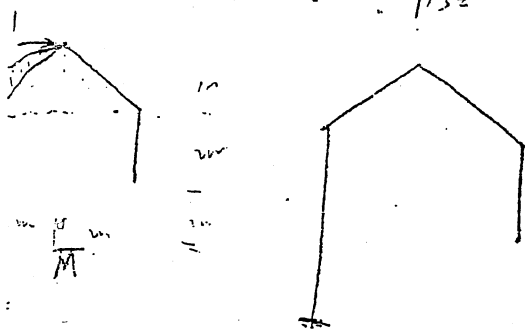
$$= -\frac{2 \cdot 20}{h} [\frac{1}{2} \times 2 \times 2 + 2 \times \sqrt{2} \times 2 + \frac{1}{2} \times 2 \times \sqrt{2} (\frac{3}{2} \times 2 + 4) + 4 \times 4] =$$

$$= -\frac{2 \cdot 20}{h} [\frac{1}{2} \times 2 \times 2 \times (\frac{3}{2} \times 2 + 4) + 2 \times \sqrt{2} \times 2 + \frac{1}{2} \times 2 \times \sqrt{2} \times (\frac{3}{2} \times 2 + 4)] =$$

$$= -\frac{2 \cdot 20}{h} [\frac{1}{2} \times 2 \times 2 \times (\frac{3}{2} \times 2 + 4) + 2 \times \sqrt{2} \times 2 + \frac{1}{2} \times 2 \times \sqrt{2} \times (\frac{3}{2} \times 2 + 4)] =$$

$$= -\frac{2 \cdot 20}{h} [1 \times 2 \times 4 + 1 \times \sqrt{2} \times 2 \times 2 + 1 \times 4 \times 4] =$$

$$\begin{aligned} X_1 + \delta_{12} X_2 + \delta_{13} X_3 + \Delta_{1t} &= 0 \\ X_1 + \delta_{22} X_2 + \delta_{23} X_3 + \Delta_{2t} &= 0 \\ X_1 + \delta_{32} X_2 + \delta_{33} X_3 + \Delta_{3t} &= 0 \end{aligned} \Rightarrow \begin{cases} X_1 = \\ X_2 = \\ X_3 = \end{cases}$$

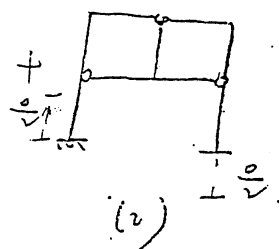
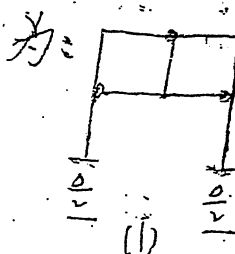


支座的位移

位移互等定理

位移互等定理

位移互等定理



(1) 位移互等定理

(2) 位移互等定理

位移互等定理

0004) 必集

由题可知 A、B 两杆为重杆

$$N_A = N_B = 0$$

$$N_D = 30 \text{ kN (压力)}$$

$$\text{取 C 点: } \sum F_y = 0 \quad N_b = N_d \cdot \frac{4}{5}$$

$$N_d = \frac{5}{4} N_b = 37.5 \text{ kN (压力)}$$

以整体为对象

$$R_B = 10 \text{ kN} \quad R_A = 40 \text{ kN}$$

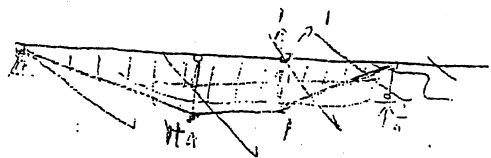
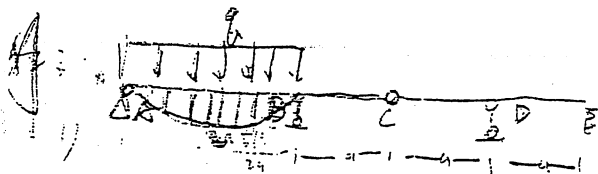
截面取成为 Z-Z, 取左面分析

$$\text{由 } \sum F_y = 0 \quad N_{cy} = 40 - 30 = 10 \text{ kN (↓)}$$

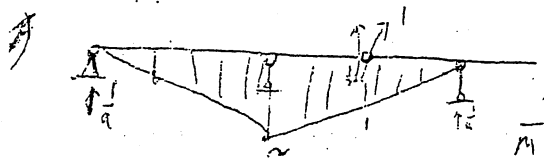
$$\text{故 } N_c = N_{cy} \cdot \frac{\sqrt{3}\pi}{4} = 10 \times \frac{\sqrt{3}}{2} = 5\sqrt{3} \text{ (压力)}$$

$$\text{另外: } N_a = 0, N_e = 0, N_b = 50 \text{ kN (压力)}$$

$$N_c = 5\sqrt{3} \text{ kN (拉)} \quad N_d = 37.5 \text{ kN (压力)}$$



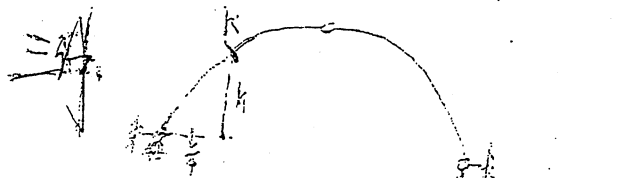
$$\Delta C = \frac{1}{EI} \left(\frac{2}{3} \cdot q \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \right) = \frac{9ql^3}{3EI} \quad (1)$$



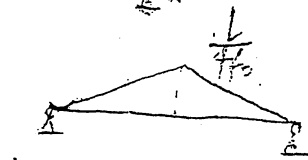
$$\Delta C = \frac{1}{EI} \left(\frac{2}{3} \cdot 2 \times \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \right) = \frac{2ql^3}{3EI} \quad (1)$$



$$\Delta C = \frac{1}{EI} \left(\frac{2}{3} \cdot q \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \right) = \frac{9ql^3}{3EI} \quad (1)$$



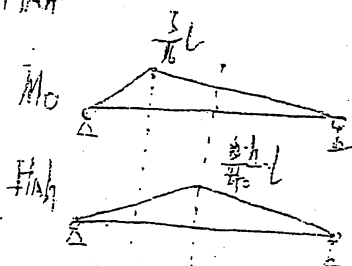
$$H_A = \frac{M_C}{f_0}$$

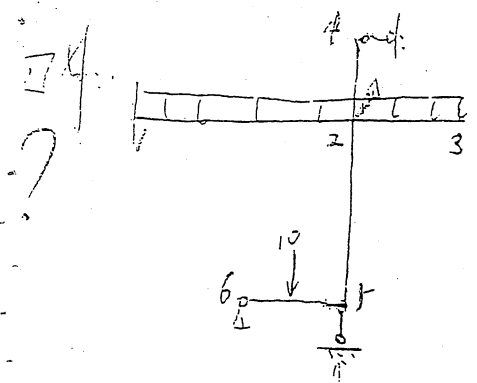


RA

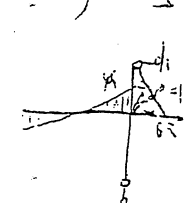


$$M_K = M_0 - H_A h$$

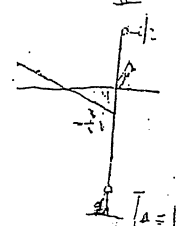
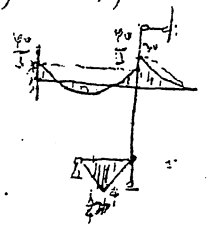




$$F_{1p} = -\frac{20}{3}, F_{2p} = (1 + 20 + \frac{91}{3}) = -41$$



$$k_{11} = 10i, k_{12} = -\frac{3}{2}i$$



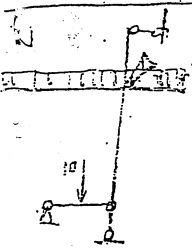
$$k_{21} = -\frac{3}{2}i, k_{22} = \frac{7}{8}i$$

$$10i\alpha_1 - \frac{3}{2}i\alpha_2 - \frac{20}{3} = 0$$

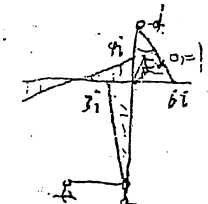
$$-\frac{3}{2}i\alpha_1 + \frac{7}{8}i\alpha_2 - 41 = 0$$

$$\alpha_1 = \frac{13.8}{i}, \alpha_2 = 87.4$$

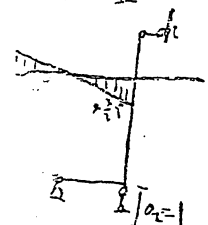
那里出错了? 因为位移计算结果处变位不平衡!



$$F_{1p} = -\frac{20}{3}, F_{2p} = -41$$



$$k_{11} = 13.7, k_{12} = -\frac{3}{2}i$$



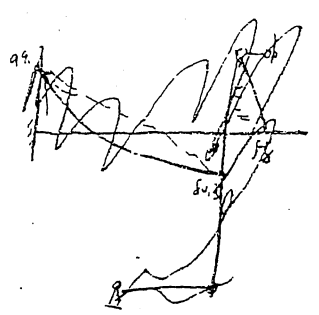
$$k_{21} = -\frac{3}{2}i, k_{22} = \frac{7}{8}i$$

$$13.7\alpha_1 - \frac{3}{2}i\alpha_2 - \frac{20}{3} = 0$$

$$-\frac{3}{2}i\alpha_1 + \frac{7}{8}i\alpha_2 - 41 = 0$$

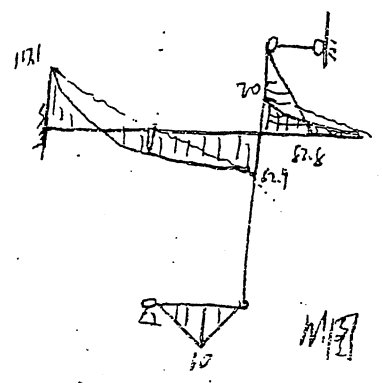
$$\alpha_1 = \frac{29}{32}, \alpha_2 = \frac{238}{32}$$

$$M = \bar{M}_1\alpha_1 + \bar{M}_2\alpha_2 + M_p$$



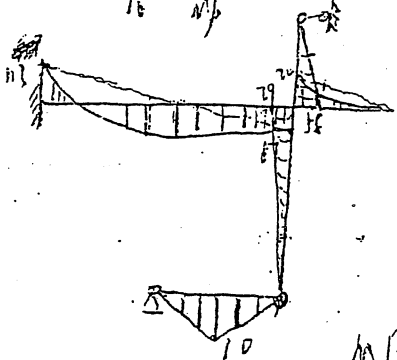
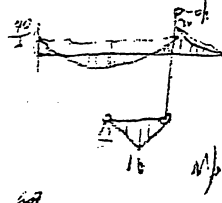
已算一种情况是已知的

已算过的, 又已知如 M_p 反时针 $M = \bar{M}_1\alpha_1 + \bar{M}_2\alpha_2 + M_p$



$$\sum M_2 = 0$$

X



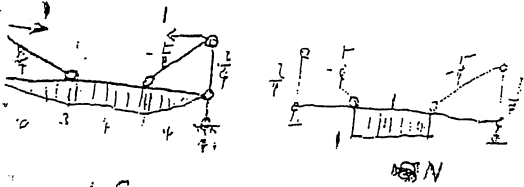
$$\sum M_2 = 0$$

✓

2001

X 2. X

$$f_c = \frac{t_1 + t_2}{2} = \frac{10 + 0}{2} = 5. \quad \sigma_t = 0 - f_c = -10 \text{ L}$$

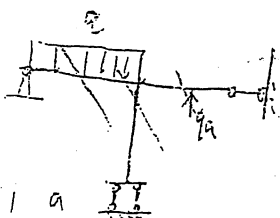
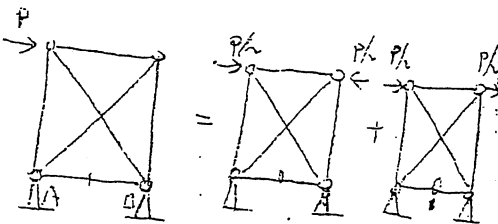


$$\begin{aligned} I &= 2t_0 \int N ds + \frac{2\sigma_t}{h} \int \bar{M} ds \\ &= 2 \cdot 5 \cdot \left(-\frac{1}{h} \times 5 \times 2 + \frac{3}{h} \times 3 \times 2 + \frac{1}{h} \times 4 \right) \\ &\quad - \frac{2 \cdot 10}{h} \cdot \left(\frac{1}{2} \times 3 \times 4 \times 2 + 3 \times 4 \right) \\ &= -602 - 4002 = -4602 \quad (2 \times 2) \end{aligned}$$

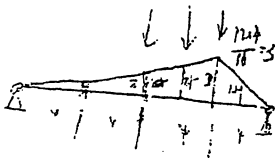
3. (X)

(X)

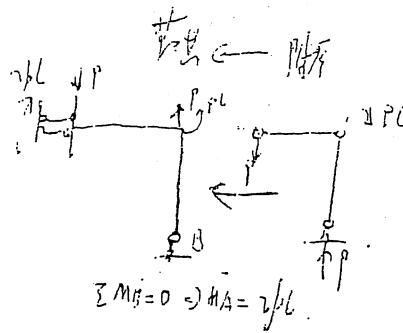
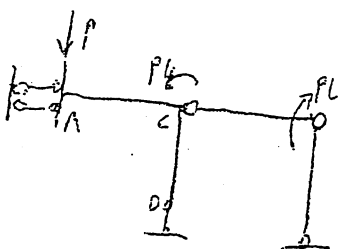
$\Delta B = 0$



(C)

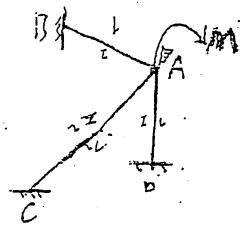


$$= 20(2 + 3 + 2 \cdot 1) = 110 \text{ kmm} \quad [10]$$



[10]

4.



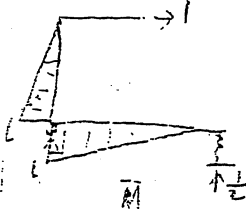
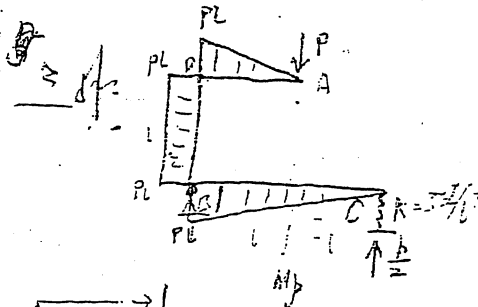
$$\dot{\theta} = \frac{\dot{\epsilon}}{L} \quad \dot{\theta}_{AD} = \dot{\theta}_A = \dot{\theta}_{AD}$$

$$M_{BA} = 2i\theta_A \quad M_{AB} = 2i\theta_B \quad M_{BC} = 2i\theta_C$$

$$4i\theta_A \times 3 = M. \Rightarrow 2i\theta_A = M/6$$

[13]

(用等截面梁法!) $\dot{\theta}_{AB} = \dot{\theta}_{AC} = \dot{\theta}_{AD}$



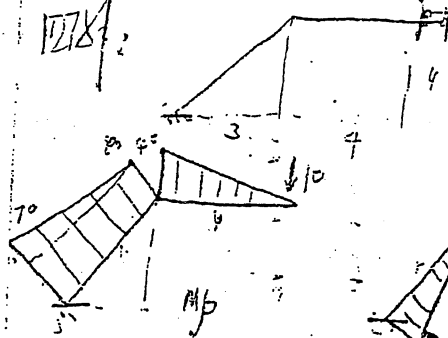
$$\Delta = \frac{1}{Ez} \left(\frac{1}{2} \times 10 \times 10 \times \frac{1}{2} \times 10 \right) - \frac{1}{Ez} \left(\frac{1}{2} \times 10 \times 10 \times \frac{1}{2} \times 10 \right)$$

$$= \frac{1}{Ez} \left(\frac{1}{2} \times 10 \times 10 \times \frac{1}{2} \times 10 + \frac{1}{2} \times 10 \times 10 \times \frac{1}{2} \times 10 \right)$$

$$= \frac{17Pl^3}{6Ez} + \frac{Pl^3}{12Ez}$$

$$= \frac{17Pl^3}{6Ez} \quad (\rightarrow)$$

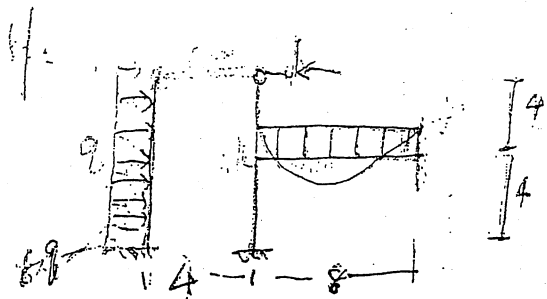
12/8



$$\delta_{22} = \frac{1}{Ez} (10 \times 10 + 10 \times 10) = \frac{9}{Ez}$$

$$\Delta \psi = -\frac{1}{Ez} \left(\frac{1}{2} \times 10 \times 10 \times \frac{1}{2} \times 10 + 10 \times 10 \times \frac{1}{2} \times 10 \right) = -\frac{31}{Ez}$$

20



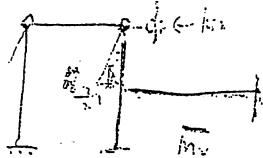
$$F_1 = -\frac{1}{1} q l^2 = -\frac{16}{3} q$$

$$F_2 = -\frac{3}{8} q l = -3.9$$



$$k_1 = \frac{52}{8} = 6.5$$

$$k_2 = -\frac{3}{2} i$$

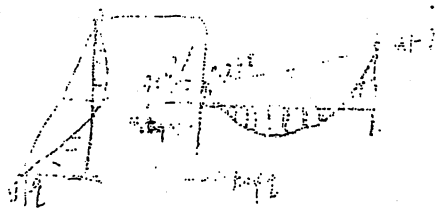


$$k_3 = -\frac{1}{2} i$$

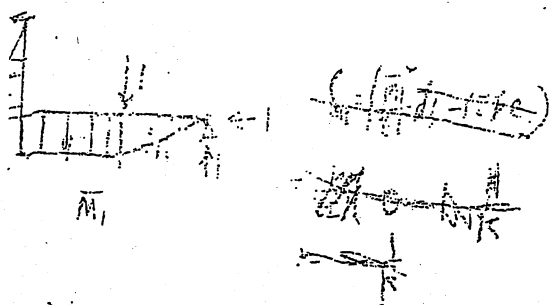
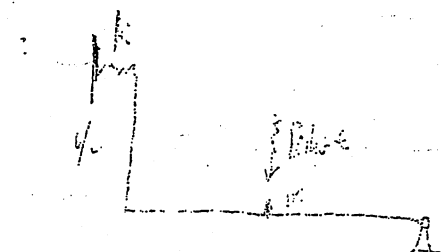
$$k_4 = \frac{27}{16} i$$

$$\begin{cases} 18i\theta_1 - \frac{3}{2}i\theta_2 - \frac{16}{3}q = 0 \\ -\frac{3}{2}i\theta_1 + \frac{27}{16}i\theta_2 - 3q = 0 \end{cases} \Rightarrow \begin{cases} \theta_1 = 1.24 \text{ rad} \\ \theta_2 = 1.16 \text{ rad} \end{cases}$$

$$M = M_1\theta_1 + M_2\theta_2 = 17.7$$



$$M = 17.7$$



$$W = \sqrt{\frac{I}{m\delta_{11}}} = \sqrt{\frac{I}{m}}$$

$$\delta_{11} = \frac{1}{EI} \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times 2 + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \right) = \frac{5}{24EI}$$

$$W = \sqrt{\frac{I}{m\delta_{11}}} = \sqrt{\frac{24EI}{5m}} = \sqrt{\frac{k}{m}}$$

$$\theta = \frac{2\pi n}{60} = \frac{2\pi \times 300}{60} = 31.4$$

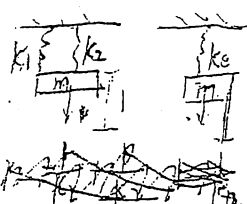
$$\beta = \frac{1}{1 - \frac{\theta^2}{\omega^2}}$$

$$y_{max} = (\beta \cdot P + m) \cdot \delta_{11} = \checkmark$$

$$W = \sqrt{\frac{k}{m}}$$

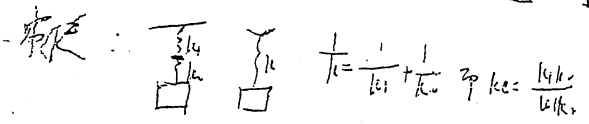
1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. 14. 15. 16. 17. 18. 19. 20. 21. 22. 23. 24. 25. 26. 27. 28. 29. 30. 31. 32. 33. 34. 35. 36. 37. 38. 39. 40. 41. 42. 43. 44. 45. 46. 47. 48. 49. 50. 51. 52. 53. 54. 55. 56. 57. 58. 59. 60. 61. 62. 63. 64. 65. 66. 67. 68. 69. 70. 71. 72. 73. 74. 75. 76. 77. 78. 79. 80. 81. 82. 83. 84. 85. 86. 87. 88. 89. 90. 91. 92. 93. 94. 95. 96. 97. 98. 99. 100. 101. 102. 103. 104. 105. 106. 107. 108. 109. 110. 111. 112. 113. 114. 115. 116. 117. 118. 119. 120. 121. 122. 123. 124. 125. 126. 127. 128. 129. 130. 131. 132. 133. 134. 135. 136. 137. 138. 139. 140. 141. 142. 143. 144. 145. 146. 147. 148. 149. 150. 151. 152. 153. 154. 155. 156. 157. 158. 159. 160. 161. 162. 163. 164. 165. 166. 167. 168. 169. 170. 171. 172. 173. 174. 175. 176. 177. 178. 179. 180. 181. 182. 183. 184. 185. 186. 187. 188. 189. 190. 191. 192. 193. 194. 195. 196. 197. 198. 199. 200. 201. 202. 203. 204. 205. 206. 207. 208. 209. 210. 211. 212. 213. 214. 215. 216. 217. 218. 219. 220. 221. 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622. 623. 624. 625. 626. 627. 628. 629. 630. 631. 632. 633. 634. 635. 636. 637. 638. 639. 640. 641. 642. 643. 644. 645. 646. 647. 648. 649. 650. 651. 652. 653. 654. 655. 656. 657. 658. 659. 660. 661. 662. 663. 664. 665. 666. 667. 668. 669. 670. 671. 672. 673. 674. 675. 676. 677. 678. 679. 680. 681. 682. 683. 684. 685. 686. 687. 688. 689. 690. 691. 692. 693. 694. 695. 696. 697. 698. 699. 700. 701. 702. 703. 704. 705. 706. 707. 708. 709. 710. 711. 712. 713. 714. 715. 716. 717. 718. 719. 720. 721. 722. 723. 724. 725. 726. 727. 728. 729. 730. 731. 732. 733. 734. 735. 736. 737. 738. 739. 740. 741. 742. 743. 744. 745. 746. 747. 748. 749. 750. 751. 752. 753. 754. 755. 756. 757. 758. 759. 760. 761. 762. 763. 764. 765. 766. 767. 768. 769. 770. 771. 772. 773. 774. 775. 776. 777. 778. 779. 780. 781. 782. 783. 784. 785. 786. 787. 788. 789. 790. 791. 792. 793. 794. 795. 796. 797. 798. 799. 800. 801. 802. 803. 804. 805. 806. 807. 808. 809. 810. 811. 812. 813. 814. 815. 816. 817. 818. 819. 820. 821. 822. 823. 824. 825. 826. 827. 828. 829. 830. 831. 832. 833. 834. 835. 836. 837. 838. 839. 840. 841. 842. 843. 844. 845. 846. 847. 848. 849. 850. 851. 852. 853. 854. 855. 856. 857. 858. 859. 860. 861. 862. 863. 864. 865. 866. 867. 868. 869. 870. 871. 872. 873. 874. 875. 876. 877. 878. 879. 880. 881. 882. 883. 884. 885. 886. 887. 888. 889. 890. 891. 892. 893. 894. 895. 896. 897. 898. 899. 900. 901. 902. 903. 904. 905. 906. 907. 908. 909. 910. 911. 912. 913. 914. 915. 916. 917. 918. 919. 920. 921. 922. 923. 924. 925. 926. 927. 928. 929. 930. 931. 932. 933. 934. 935. 936. 937. 938. 939. 940. 941. 942. 943. 944. 945. 946. 947. 948. 949. 950. 951. 952. 953. 954. 955. 956. 957. 958. 959. 960. 961. 962. 963. 964. 965. 966. 967. 968. 969. 970. 971. 972. 973. 974. 975. 976. 977. 978. 979. 980. 981. 982. 983. 984. 985. 986. 987. 988. 989. 990. 991. 992. 993. 994. 995. 996. 997. 998. 999. 1000. 1001. 1002. 1003. 1004. 1005. 1006. 1007. 1008. 1009. 1010. 1011. 1012. 1013. 1014. 1015. 1016. 1017. 1018. 1019. 1020. 1021. 1022. 1023. 1024. 1025. 1026. 1027. 1028. 1029. 1030. 1031. 1032. 1033. 1034. 1035. 1036. 1037. 1038. 1039. 1040. 1041. 1042. 1043. 1044. 1045. 1046. 1047. 1048. 1049. 1050. 1051. 1052. 1053. 1054. 1055. 1056. 1057. 1058. 1059. 1060. 1061. 1062. 1063. 1064. 1065. 1066. 1067. 1068. 1069. 1070. 1071. 1072. 1073. 1074. 1075. 1076. 1077. 1078. 1079. 1080. 1081. 1082. 1083. 1084. 1085. 1086. 1087. 1088. 1089. 1090. 1091. 1092. 1093. 1094. 1095. 1096. 1097. 1098. 1099. 1100. 1101. 1102. 1103. 1104. 1105. 1106. 1107. 1108. 1109. 1110. 1111. 1112. 1113. 1114. 1115. 1116. 1117. 1118. 1119. 1120. 1121. 1122. 1123. 1124. 1125. 1126. 1127. 1128. 1129. 1130. 1131. 1132. 1133. 1134. 1135. 1136. 1137. 1138. 1139. 1140. 1141. 1142. 1143. 1144. 1145. 1146. 1147. 1148. 1149. 1150. 1151. 1152. 1153. 1154. 1155. 1156. 1157. 1158. 1159. 1160. 1161. 1162. 1163. 1164. 1165. 1166. 1167. 1168. 1169. 1170. 1171. 1172. 1173. 1174. 1175. 1176. 1177. 1178. 1179. 1180. 1181. 1182. 1183. 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1848. 1849. 1850. 1851. 1852. 1853. 1854. 1855. 1856. 1857. 1858. 1859. 1860. 1861. 1862. 1863. 1864. 1865. 1866. 1867. 1868. 1869. 1870. 1871. 1872. 1873. 1874. 1875. 1876. 1877. 1878. 1879. 1880. 1881. 1882. 1883. 1884. 1885. 1886. 1887. 1888. 1889. 1890. 1891. 1892. 1893. 1894. 1895. 1896. 1897. 1898. 1899. 1900. 1901. 1902. 1903. 1904. 1905. 1906. 1907. 1908. 1909. 1910. 1911. 1912. 1913. 1914. 1915. 1916. 1917. 1918. 1919. 1920. 1921. 1922. 1923. 1924. 1925. 1926. 192

2002

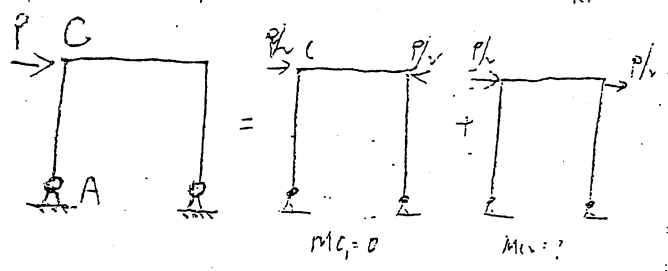


并联: $k_e = k_1 + k_2$

[A]



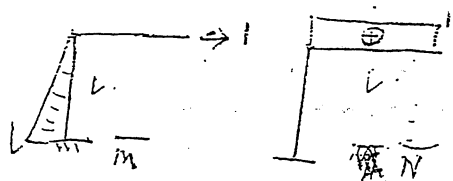
$\frac{1}{k_e} = \frac{1}{k_1} + \frac{1}{k_2}$ $\Rightarrow k_e = \frac{k_1 k_2}{k_1 + k_2}$



$\frac{1}{2} M_A = P \cdot h$

[C]

4. $t_0 = \frac{3}{2} t$ $at = t - t_0 = -t$

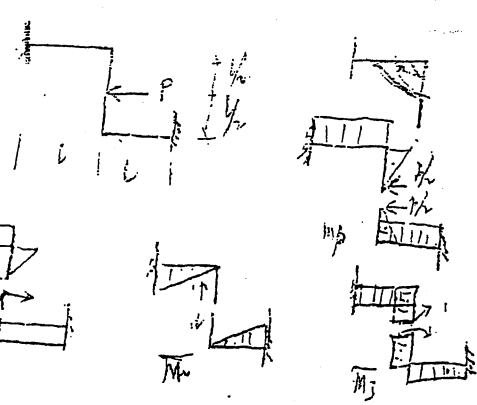


$\Delta_{CH} = 2 \cdot t_0 \cdot \int \bar{N} dt + \frac{2 \cdot at}{h} \cdot \int \bar{N} ds$

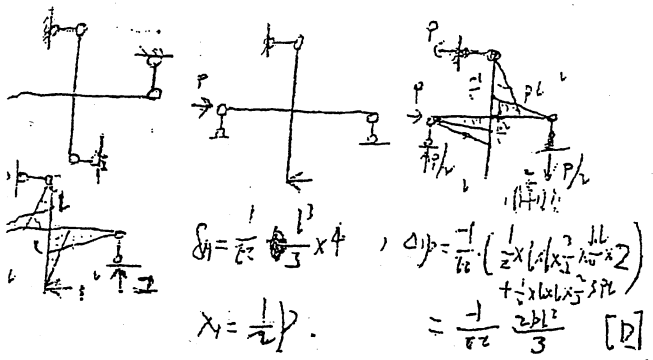
$= 2 \cdot \frac{3}{2} t \times l \times l + 2 \cdot t \times \frac{1}{2} \times \frac{1}{2} \times l$

$= \frac{3}{2} \cdot 2 \cdot t \cdot l + \frac{1}{2} \cdot 2 \cdot t \cdot l$

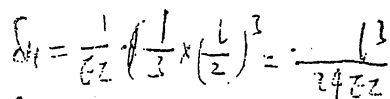
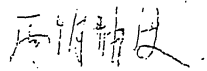
(X)



$\Delta_1 = 0, \Delta_2 = 0, \Delta_3 = 0$ [D]



$\Delta_1 = \frac{1}{E} \cdot \frac{1}{3} \times 4$ $\Delta_2 = \frac{1}{E} \cdot \left(\frac{1}{2} \times l \times \frac{1}{2} \times \frac{1}{2} \times 2 \right)$ $\Delta_3 = \frac{1}{E} \cdot \frac{2 \cdot l^2}{3}$



$$S_{12} = S_{21} = \frac{1}{Z_0} \left[\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \left(\frac{1}{4} + \frac{2}{1} \times \frac{1}{4} \right) \right] = \frac{1}{96Z_0}$$

$$u = \frac{1}{62} \cdot \left(\frac{1}{2} \times \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \right) + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \right)$$

$$\frac{1}{\omega} \left| \begin{array}{cc} m_1 \omega_1 - \frac{1}{\omega} & m_2 \omega_2 \\ m_1 \omega_1 & m_2 \omega_2 - \frac{1}{\omega} \end{array} \right| = 0$$

7/10

$$\frac{1}{\lambda} = \frac{1}{w^2}$$

$$\lambda_i = \frac{(m_1 \delta_{11} + m_2 \delta_{22}) \pm \sqrt{(m_1 \delta_{11} + m_2 \delta_{22})^2 - 4 m_1 m_2 (\delta_{11} \delta_{22} - \delta_{12}^2)}}{2}$$

$$= \frac{m^3}{9.62} (T \pm 57) = \frac{0.1317 \frac{m^3}{\text{g}}}{0.0121 \frac{m^3}{\text{g}}} =$$

$$W_{\text{eff}} = 7.4794 \frac{\text{Z}}{\text{m}^2}$$

$$W_2 = \frac{82.11872}{\ln 2}$$

$$W_1 = 2.75 \sqrt{\frac{C_2}{n_1 \beta}}$$

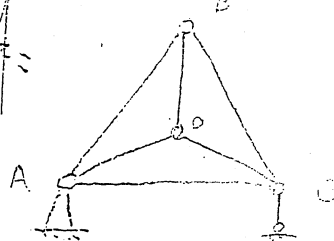
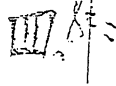
$$W_v = 9.062 \sqrt{\frac{\tau \cdot \tau}{m \cdot v}}$$

$$(M_{12} - \frac{1}{v}) Y_1 + M_{22} Y_2 = 0$$

$$M_{2001} Y_c + (M_{2001} - \frac{1}{w_c}) Y_h = 0$$

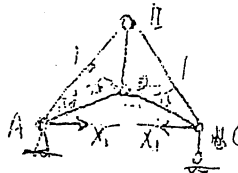
$$\Rightarrow \frac{Y_1}{Y_2} = \frac{-m_2 \delta_{12}}{m_1 \delta_{11} - \frac{1}{w_1}} = \frac{0.1559}{1}$$

$$\frac{Y_{12}}{Y_{11}} = \frac{-A_{12} \delta_{12}}{A_{11} \delta_{11} - \frac{1}{k_{12}}} = \frac{-1.7662}{1}$$



$$h=6 \quad n=2+2+2+1=8 \quad C_0 = 1$$

$$W = 18 - 16 - 3 = -1 \quad \text{一次超静定结构}$$



$$N_{AD} \cos 30^\circ = 1 + N_{AB} \sin 30^\circ$$

$$\overline{N_{AB}} \cdot \Omega_{AB}^0 = \overline{N_{AJ}} \cdot \Omega_{AJ}^0$$

$$\Rightarrow \begin{cases} \overline{N_{AB}} = \sqrt{2} & (\text{E}) \\ \overline{M_{AB}} = 1 & (\text{拉}) \end{cases}$$

$$N_{BD} = -\sqrt{3} \text{ (E)}$$

17/12 $N_{100} = \sqrt{3} \text{ (7)}$
 $N_{101} = 1 \text{ (12)}$

$$N_{ac} = 1 \text{ (粒)}$$

$$\delta_{II} = \frac{\sum \overline{M}^2 L}{EA} = \frac{1}{EA} \left(1 \times 3 + 3 \times \frac{\sqrt{3}}{3} \times 3 \right) = \frac{4\sqrt{3}}{EA}$$

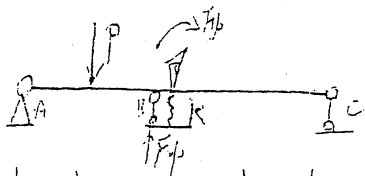
$$\sum_{i=1}^n x_i + \Delta_{3L} = 0$$

$$\Delta t = 2t \int \sqrt{u} ds = 2t \left[-\sqrt{u} \frac{3}{2} \ln x \right]$$

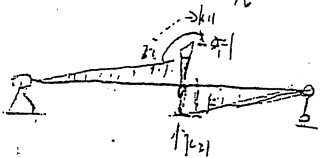
$$X_1 = \frac{32tL}{\frac{3EI}{EA}L} = \frac{2tEA}{1+\sqrt{2}}$$

$$N_{AB} = 1 \cdot X_1 = \frac{2 + \sqrt{17}}{14\sqrt{3}} \div N_{BC} = N_{AC}$$

$$N_{AD} = N_{BD} = N_{CD} = -\frac{\sqrt{3}}{1+\sqrt{3}} 2tEA$$



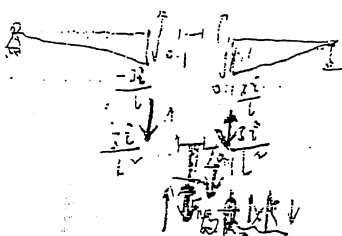
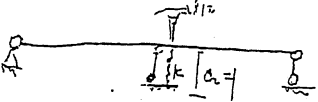
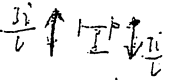
$M_{BA} = \frac{3}{16}Pl$
 $Q_{BA} = -\frac{11}{16}Pl$
 $\Rightarrow F_b = \frac{3}{16}Pl$
 $F_b = -\frac{11}{16}Pl$



$$\frac{EI}{L} = i$$

$$k_{11} = 6i$$

$$k_{21} = 0$$



$$k_{11} = 0$$

$$k_{21} = 0$$

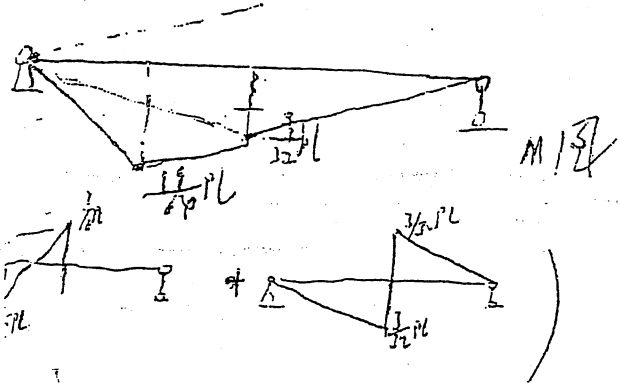
$$k_{22} = \frac{4i}{L} + \frac{16k}{L} = \frac{16i}{L}$$

$$k_{11} \Delta_1 + k_{12} \Delta_2 + F_{1p} = 0$$

$$k_{21} \Delta_1 + k_{22} \Delta_2 + F_{2p} = 0$$

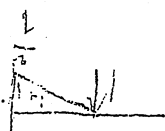
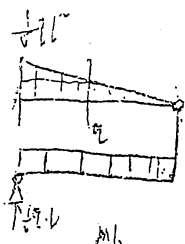
$$\begin{cases} \frac{3}{16}Pl + \frac{3}{16}Pl = 0 \\ \frac{11}{16}Pl + \frac{11}{16}Pl = 0 \end{cases} \Rightarrow \begin{cases} \Delta_1 = -\frac{1}{32}Pl \\ \Delta_2 = +\frac{1}{16}Pl \end{cases}$$

$$M = \bar{M}_1 \Delta_1 + \bar{M}_2 \Delta_2 + M_p$$



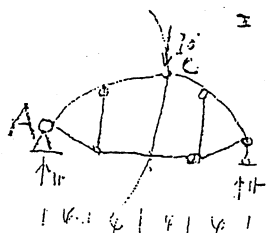
2003

7. [D]



(X)

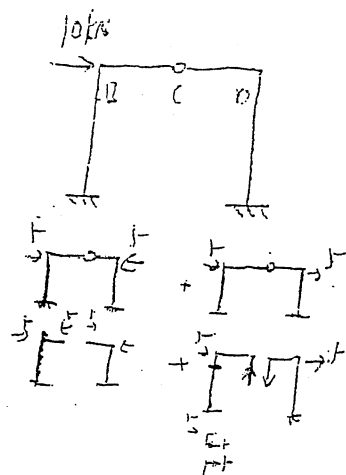
$$\Delta = \frac{1}{EI} \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \left(\frac{10}{3} + \frac{10}{3} \times \frac{10}{10} \right) \right)$$



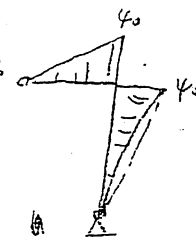
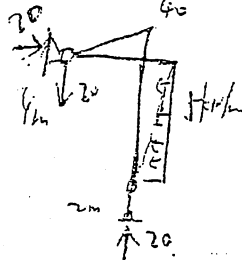
$$\begin{aligned} \sum M_A &= 0 \\ N_{DE} \times 6 &= 15 \times 8 \\ N_{DE} &= 20 \end{aligned}$$

(X)

(X)



设计钢筋混凝土



1. [D] 3 > 1.0 无振动的

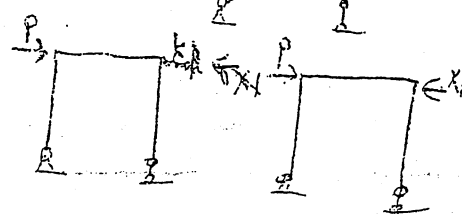
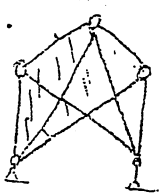
2. [D] $\frac{12EI}{L^3} = \frac{4EI}{L^3}$

[]

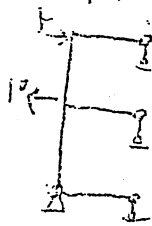
[B] $W = 3.8 - 2.1 - 3 = -1$

角位移

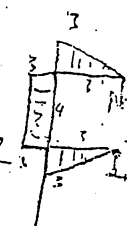
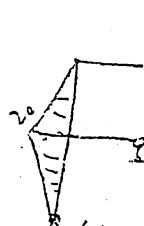
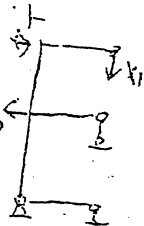
虚功原理



取半片



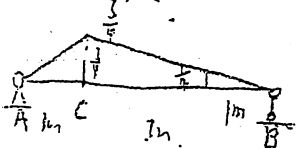
取半片



$$X_1 = \frac{-\Delta_p}{\delta_{11}} = -\frac{\frac{1}{EI} \left(\frac{1}{2} \times 4 \times 2 \times 3 \right)}{\frac{1}{EI} \left(\frac{1}{3} \times 3 \times 2 + 3 \times 3 \right)} = -\frac{20}{9}$$

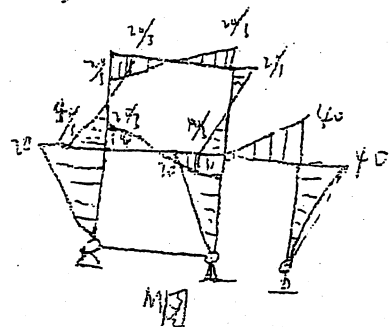
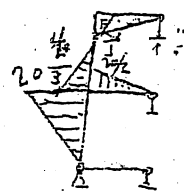
$$M = \overline{M}_1 X_1 + M_p$$

[B]

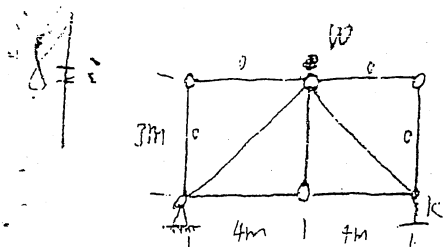


$$M = 5 \times \frac{1}{2} \times 2 \times \frac{1}{3} = 10.7$$

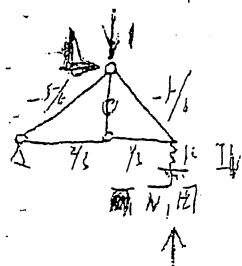
$$M = 28 \quad m = 28 \times \frac{3}{4} = 21$$



25

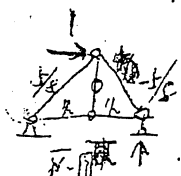


求位移



$$\delta u_1 = \sum \frac{\bar{F}}{EA} L = \bar{F} \cdot \bar{\Delta} = \frac{1}{EA} \left(\frac{1}{2} \times 2 \times 2 + \frac{1}{2} \times 2 \times 2 \right) + \frac{1}{EA} \times \frac{1}{2} \times 2 \times 2$$

$$= \frac{2}{3} \times \frac{1}{EA} = \frac{2}{3EA}$$



$$\delta u_2 = \frac{1}{EA} \left(\frac{1}{2} \times 2 \times 2 + \frac{1}{2} \times 2 \times 2 \right) + \frac{1}{EA} \times \frac{1}{2} \times 2 \times 2$$

$$= \frac{2}{3} \times \frac{1}{EA}$$

$$\delta u = \sum \frac{\bar{F}}{EA} L = \bar{F} \cdot \bar{\Delta} = \frac{1}{EA} \left(-\frac{1}{2} \times 2 \times 2 + \frac{1}{2} \times 2 \times 2 + \frac{1}{2} \times 2 \times 2 \right) + \frac{1}{EA} \times \frac{1}{2} \times 2 \times 2$$

$$= \frac{1}{EA} \times \frac{1}{2} \times 2 \times 2 = \frac{1}{EA}$$

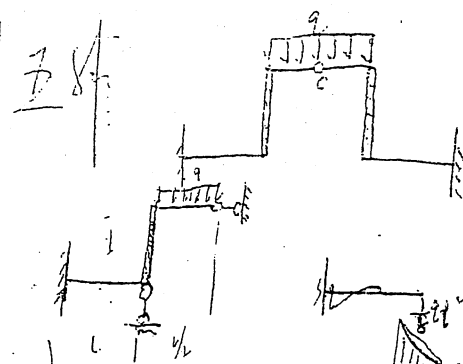
$$\lambda = \frac{1}{w^2}$$

$$\lambda_L = \frac{(M_1 \delta u_1 + M_2 \delta u_2) \pm \sqrt{(M_1 \delta u_1 + M_2 \delta u_2)^2 - 4 M_1 M_2 (\delta u_1 \delta u_2)}}{2}$$

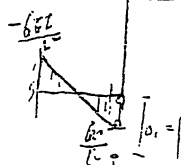
=

$$w_1 =$$

$$w_2 =$$

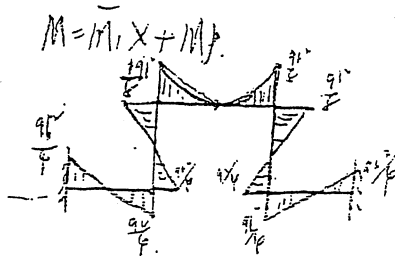


$$F_y = -\frac{1}{2} q L$$



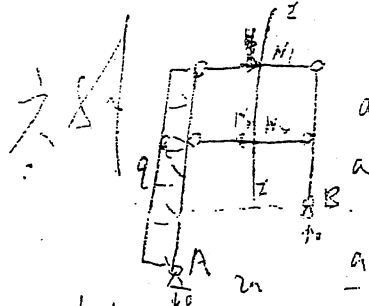
$$k_{11} = \frac{12EI}{L^3}$$

$$k_{11} \cdot \Delta_1 + F_y = 0 \Rightarrow \Delta_1 = \frac{q L^4}{12EI}$$



M/EI

(可用力法求解!)



取断两杆取右半段，由 $\sum F_y = 0 \Rightarrow V_E = 0$

$$\text{由 } \sum M_B = 0: N_1 \times 2a + N_2 \times a = 0 \quad (1)$$

取左半段，由 $\sum M_B = 0$

$$\frac{1}{2} q (3a)^2 + N_1 \times 3a = N_2 \times 2a \quad (2)$$

联立 (1) (2)

$$\Rightarrow N_1 = \frac{9}{2} qa \text{ (拉力)}$$

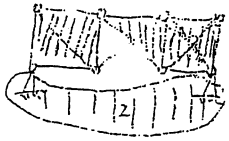
$$N_2 = 9qa \text{ (压力)}$$

204

X) 2: (0) 3(0)

$$W = 3 \times 11 - 7 \times 4 - 6 = 33 - 28 - 6 = -1 \text{ 顺时针}$$

图 1 所示



1/3 处

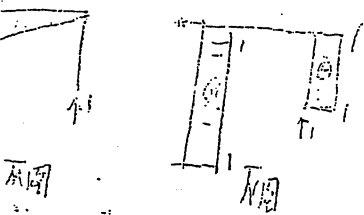
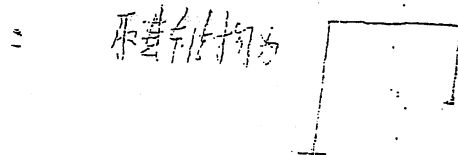
[B]

[B]

[A]

[C]

[C]



$$= \frac{t_1 + t_2}{2} = \frac{20 - 10}{2} = 5^\circ$$

$$= t_1 - t_2 = 20 - 10 = 10^\circ$$

$$= 2t_0 \int \bar{N} ds + \frac{2t_0}{h} \int \bar{M} ds$$

$$= 2 \times 5 \cdot (1 \times 1 - \frac{1}{2}) + \frac{2 \times 30}{1/10} \cdot (\frac{1}{2} \times 1 \times 1 + 1 \times 1)$$

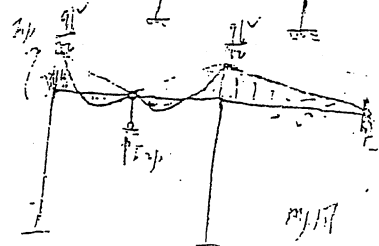
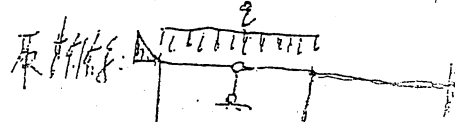
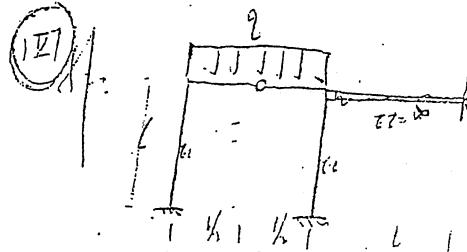
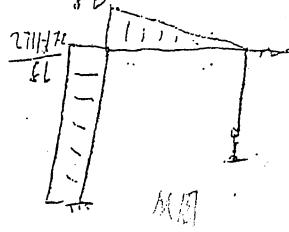
$$= \frac{5}{2} \times 2 \cdot 1 + 5 \times 90 \times 2 \cdot 1 = 412.5 \times 1$$

$$\frac{1}{2} \cdot (\frac{1}{3} l^3 + 1 \times 1) = \frac{4 l^3}{362}$$

$$\text{由 } \delta_{11} X_1 + \delta_{12} = 0$$

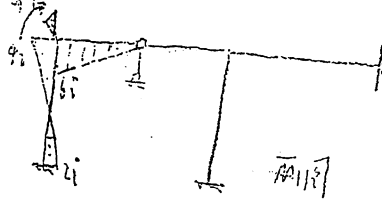
7.07

$$X_1 = \frac{-\frac{412.5 l^3}{362}}{4 l^3} \cdot 3 E I = \frac{711.25 E I}{8 l^3}$$

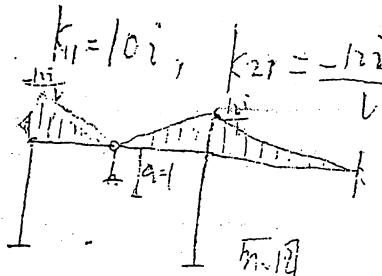


$$F_y = -\frac{1}{2} q \left(\frac{L}{2}\right)^2 = -\frac{9 l^2}{32}$$

$$F_y = -\left(\frac{3}{16} q \left(\frac{L}{2}\right) + \frac{1}{16} q L\right) = -\frac{1}{2} q L$$

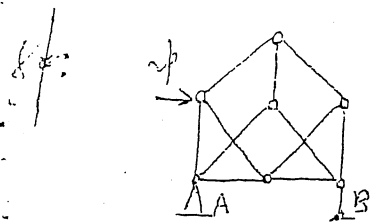


$$\bar{M} = \frac{63}{L}$$



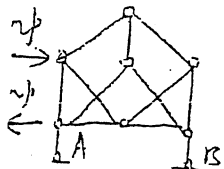
$$k_{11} = 10i, \quad k_{21} = -\frac{12i}{L}$$

$$k_{12} = -\frac{12i}{L}, \quad k_{22} = \frac{48i}{L^3}$$



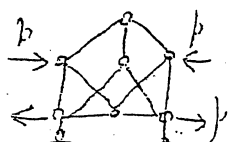
桁架结构

由节点平衡条件可得如下方程组

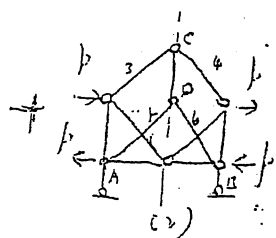


即将A点平衡条件
并代入之求各杆内力
若载P。

桁架结构为对称桁架，然后将荷载化为



(1)



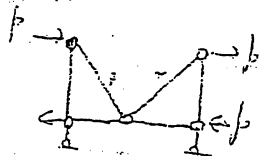
桁架结构的内力

桁架结构的内力

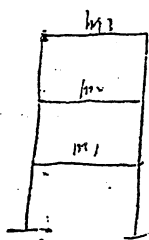
下， $N_1 = N_2 = 0$ 。

下，CD杆位于中轴线上， $N_{CD} = 0$ ，并使桁架的上下

桁架结构的内力



由桁架结构的内力 $N_1 = -P/2$ ， $N_2 = P/2$ (拉)



由桁架结构的自由振动的特征方程

$$[-\omega^2 M] \{Y\} = 0 \quad l+m$$

$L(n, m, n, m)$

$$Y_3 = \begin{pmatrix} a \\ b \\ 1 \end{pmatrix}$$

$$C \begin{pmatrix} 1/2 & 2/3 & 1 \end{pmatrix} \begin{pmatrix} 270 & 270 & 180 \end{pmatrix} \begin{pmatrix} a \\ b \\ 1 \end{pmatrix} = 0$$

$$a + 2b + 2 = 0 \quad (1)$$

$$C \begin{pmatrix} -2/3 & -2/3 & 1 \end{pmatrix} \begin{pmatrix} 270 & 270 & 180 \end{pmatrix} \begin{pmatrix} a \\ b \\ 1 \end{pmatrix} = 0$$

$$a + b - 1 = 0 \quad (2)$$

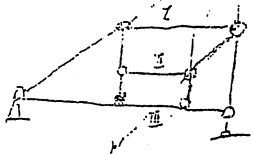
$$\text{由 (1) (2) } \Rightarrow \begin{cases} a = 4 \\ b = -3 \end{cases}$$

$$Y_3 = \begin{pmatrix} 4 \\ -3 \\ 1 \end{pmatrix}$$

2007 181 (3 分)

$$m=9, n=12, c=3$$

$$W=27-12-3 \leq 0$$



181 (3 分)

(X)



[0]

[0]

[C]

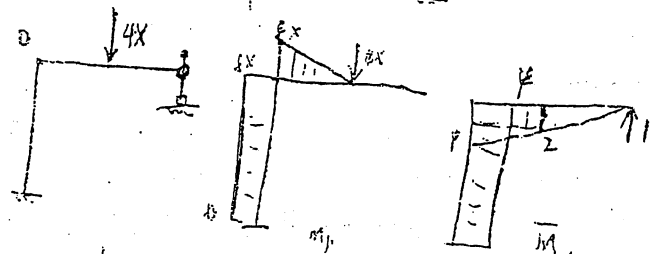
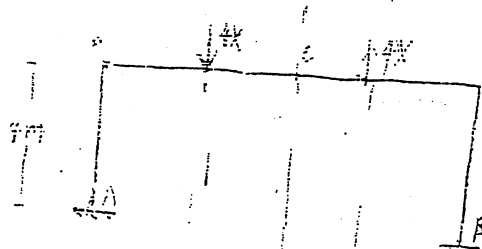
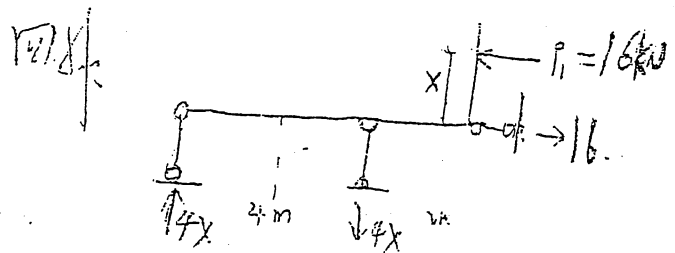
[D]

[D]

[C]

[B]

$$4x - \frac{1}{2} \times 4 \times 4 + 7.2x = 0$$



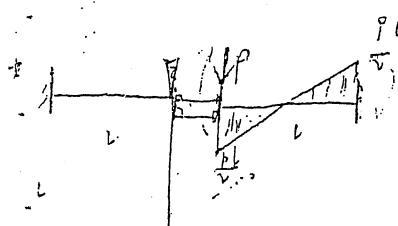
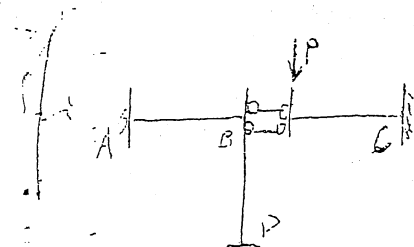
$$\delta_{11} = \frac{1}{EI} \left(\frac{1}{2} \times 4 \times 4 + \frac{1}{2} \times 4 \times 4 + 4 \times 4 \times 4 \right) = \frac{64x^4}{3EI}$$

$$\delta_{12} = -\frac{1}{EI} \left(4 \times 4 \times 4 + \frac{1}{2} \times 4 \times 4 \times (2 + \frac{2}{3}) \right) = -\frac{46x^4}{3EI}$$

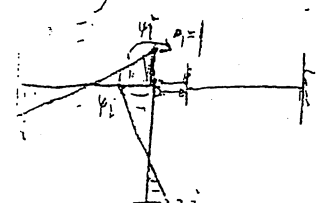
$$X_1 = -\frac{\delta_{12}}{\delta_{11}} = \frac{46x^4}{64x^4} = 0.71875$$

$$M_b = 8x - 7.2x = 0.8x = 2.25 \text{ kN} \cdot \text{m}$$

$$x = 3 \text{ m}$$



$$F_{1P} = \frac{Pl}{2}$$

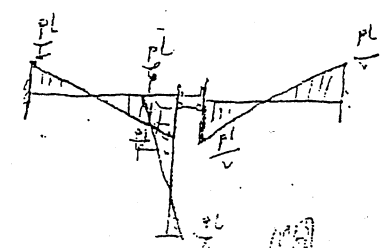


$$i = \frac{EI}{L}$$

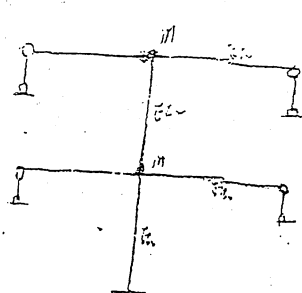
$$k_{11} = 8i$$

$$F_{10} + F_{1P} = 0, \quad \Delta_{1P} = -\frac{Pl}{16i}$$

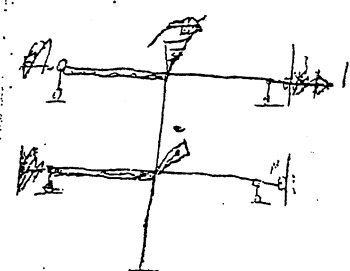
$$M = \bar{M}_1 + M_P$$



六八

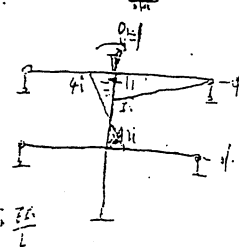


两向位移



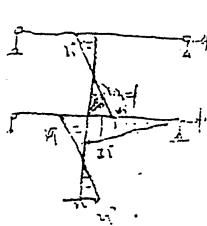
$$F_{3P} = -1$$

$$F_{1P} = 0 = F_{2P} = F_{4P}$$



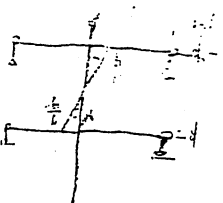
$$k_{11} = 7i, \quad k_{12} = 2i$$

$$k_{13} = -\frac{6i}{L}, \quad k_{14} = \frac{6i}{L}$$



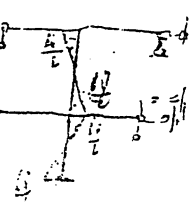
$$k_{11} = 7i, \quad k_{12} = 11i$$

$$k_{13} = -\frac{6i}{L}, \quad k_{14} = \frac{6i}{L}$$



$$k_{17} = -\frac{6i}{L} = k_{13}, \quad k_{123} = -\frac{6i}{L}$$

$$k_{133} = \frac{12i}{L^2}, \quad k_{172} = -\frac{62i}{L^2}$$



$$k_{14} = \frac{6i}{L} = k_{11}$$

$$k_{24} = \frac{6i}{L} = k_{12}$$

$$k_{34} = -\frac{12i}{L^2} = k_{13}$$

$$k_{44} = \frac{24i}{L^2}$$

2006.

1. 判断题

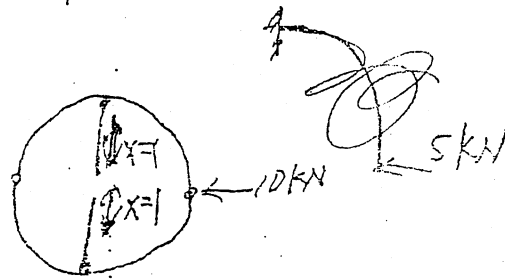
1. X 2. X 3. X 4. V 5. V 6. V

2. 填空题

1. $\frac{2EI}{h}$ 2. 1.12 cm 3. 90 kN·m

4. $\sqrt{\frac{k_a + k_b}{\rho m}}$

三. 计算题



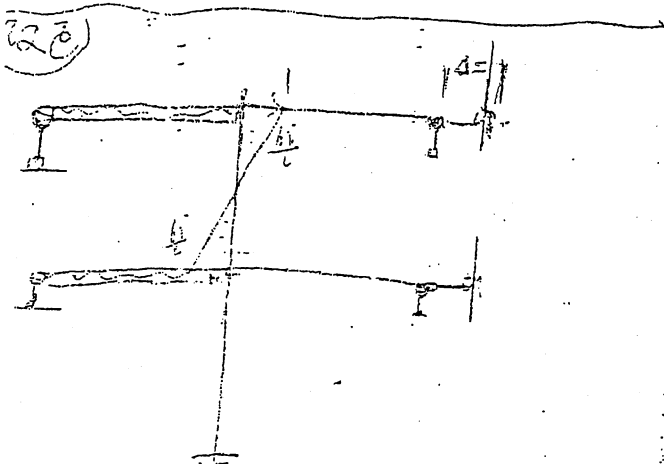
$$4_1 + k_{11}\Delta_1 + k_{12}\Delta_2 + k_{13}\Delta_3 + \bar{F}_1 = 0$$

$$k_{21}\Delta_1 + k_{22}\Delta_2 + k_{23}\Delta_3 + \bar{F}_2 = 0$$

$$k_{31}\Delta_1 + k_{32}\Delta_2 + k_{33}\Delta_3 + \bar{F}_3 = 0$$

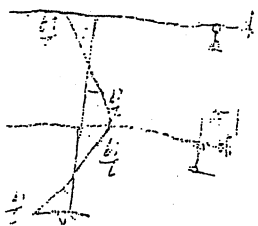
$$k_{41}\Delta_1 + k_{42}\Delta_2 + k_{43}\Delta_3 + \bar{F}_4 = 0$$

$$\begin{cases} \Delta_1 = \\ \Delta_2 = \\ \Delta_3 = \\ \Delta_4 = \end{cases}$$



$$\bar{F}_1 = -1, \bar{F}_2 = 0$$

$$k_{11} = \frac{12EI}{l^3}, k_{12} = -\frac{6EI}{l^2}$$



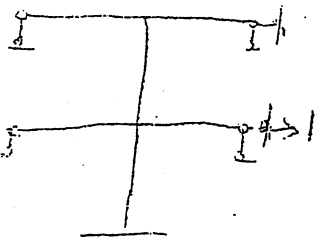
$$k_{12} = -\frac{6EI}{l^2}$$

$$k_{22} = \frac{4EI}{l}$$

$$k_{11}\Delta_1 + k_{12}\Delta_2 + \bar{F}_1 = 0 \Rightarrow \Delta_1 = \checkmark$$

$$k_{12}\Delta_1 + k_{22}\Delta_2 + \bar{F}_2 = 0 \Rightarrow \Delta_2 = \checkmark$$

$$M_f = M_1\Delta_1 + M_2\Delta_2 + M_3$$



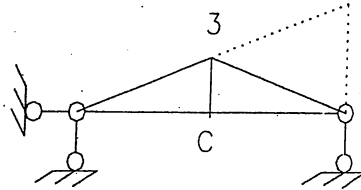
$$M_2 =$$

$$\frac{1}{2} \left(\frac{1}{18} \delta_{11} + \frac{1}{18} \delta_{22} + \frac{1}{18} \delta_{33} \right) \lambda_c = \frac{w_c}{w_c}$$

天津大学 2008 年结构力学真题参考答案

一、

1. 错误，反力互等定理反力系数数值相等，量纲也相同。
2. 错误，承受动力荷载的结构位移不仅由结构刚度决定，还包括动力荷载的频率等因素。
3. 正确，绘出 M_C 影响线如图所示， $10kN$ 在 C 点时， $M_C = \frac{85}{2} kN\cdot m$ ； $5kN$ 在 C 点时， $M_C = 25kN\cdot m$ ； $3kN$ 在 C 点时， $M_C = 41kN\cdot m$ ； $7kN$ 在 C 点时， $M_C = 24kN\cdot m$ 。

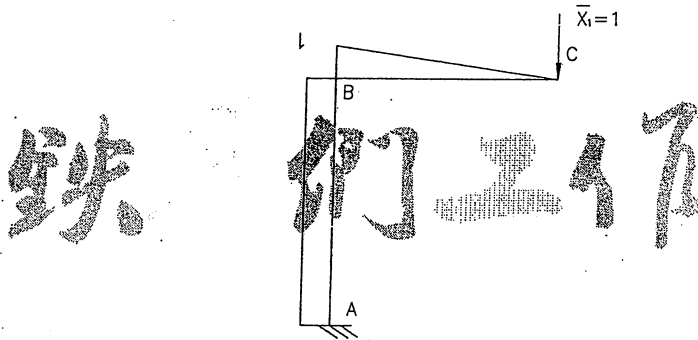


4. 正确。
5. 错误，在基本结构受竖向荷载时，轴向力为零，轴向多余未知力也就为零。

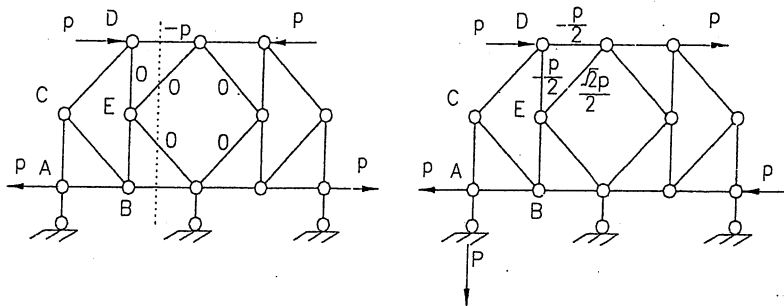
二、

1. 1；在阴影部分的角部加一个刚臂，各点位移就可以确定。
2. $61.85kN\cdot m$ ， $M = \overline{M}_1 X_1$ ， $\overline{M}_1$ 不变；

$$\delta_{11} X_1 + \Delta_{1t} = 0, \delta_{11} = \frac{4l^3}{3EI}, \Delta_{1t} = \frac{\alpha \Delta t}{h} \frac{3l^2}{2}, X_1 = -\frac{9EI\alpha\Delta t}{8hl}.$$



3. 几何不变体系，有一个多余约束，利用增加二元体的方法，HI 为多余约束。



4. $-\frac{P}{2}$

5. 从大变小, $\delta_{11}X_1 + \Delta_{1P} = -\frac{1}{f}, X_1 = -\frac{\frac{1}{f} + \Delta_{1P}}{\delta_{11}}, M = \overline{M}_1X_1 + M_P, \overline{M}_1$ 和 M_P 不变。

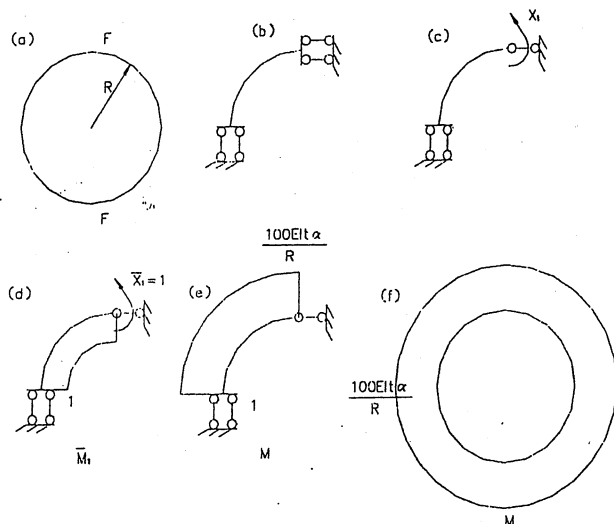
三、

结构荷载有两个对称轴，取 $\frac{1}{4}$ 结构进行分析，典型方程 $\delta_{11}X_1 + \Delta_{1t} = 0$,

$$\delta_{11} = \frac{1}{EI} \int_0^{\frac{\pi}{2}} R d\theta, \Delta_{1t} = \frac{\alpha \Delta t}{h} A_{\overline{w}_M} = \frac{10\alpha \Delta t}{R} \int_0^{\frac{\pi}{2}} R d\theta = \frac{100\alpha}{R} \frac{\pi R}{2}$$

$$X_1 = -\frac{100EI\alpha}{R}$$

天大考 QQ:1264855369
研资料 tel:15122905501



利用 $M = \overline{M}_1X_1$ 绘出弯矩图，再利用对称性将整体结构的弯矩图绘出。

四、

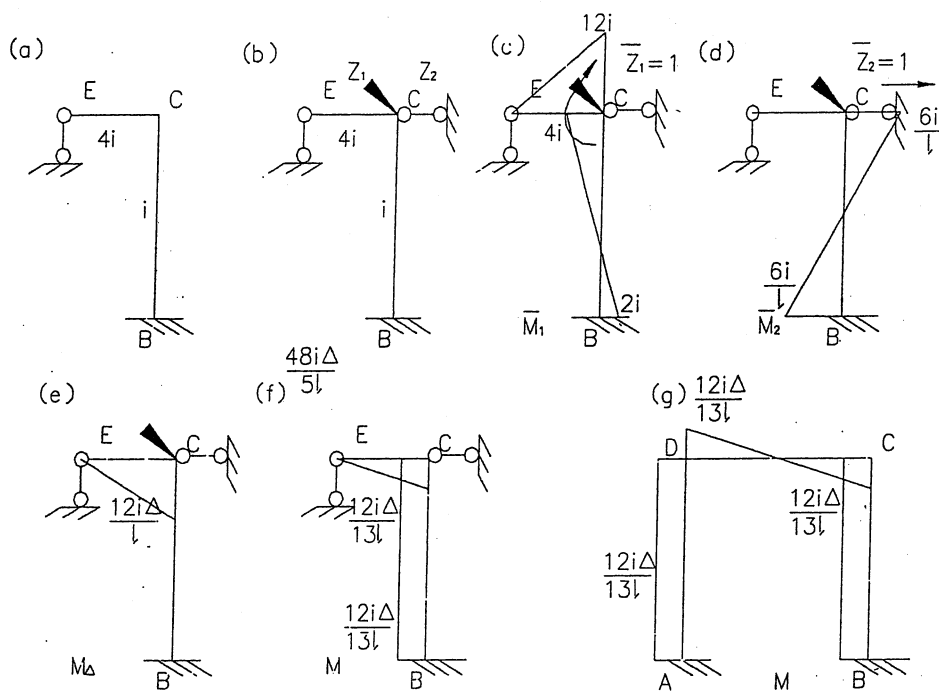
将位移荷载分为两组，一组为 AB 同时下移 $\frac{\Delta}{2}$ ，此时不产生弯矩，另一组为 A 上移 $\frac{\Delta}{2}$ ，B 下移 $\frac{\Delta}{2}$ ，

此时结构荷载反对称，取半结构。典型方程
$$\begin{cases} r_{11}Z_1 + r_{12}Z_2 + R_{1\Delta} = 0 \\ r_{21}Z_1 + r_{22}Z_2 + R_{2\Delta} = 0 \end{cases}$$

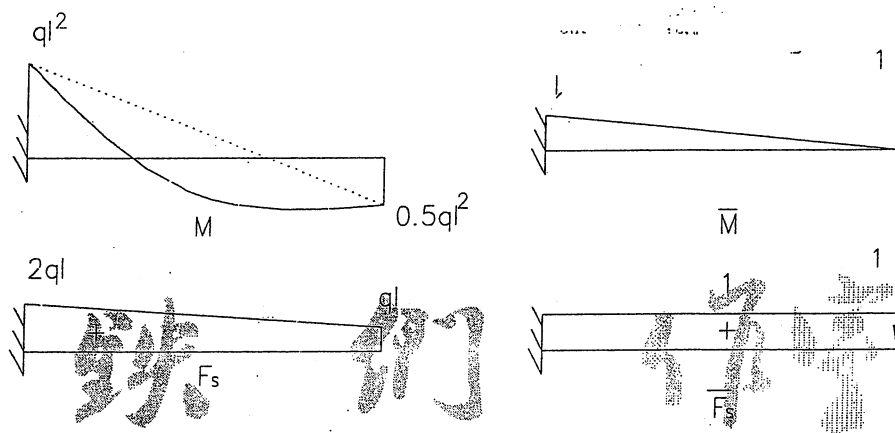
$$r_{11} = 16i, r_{12} = r_{21} = -\frac{6i}{l}, r_{22} = \frac{12i}{l^2}, R_{1\Delta} = -\frac{12i\Delta}{l}, R_{2\Delta} = 0$$

$$Z_1 = \frac{12\Delta}{13l}, Z_2 = \frac{6\Delta}{13}$$

由 $M = \overline{M}_1Z_1 + \overline{M}_2Z_2 + M_{\Delta}$ 绘出弯矩图，最后由结构荷载的对称性得出整体结构的弯矩图。



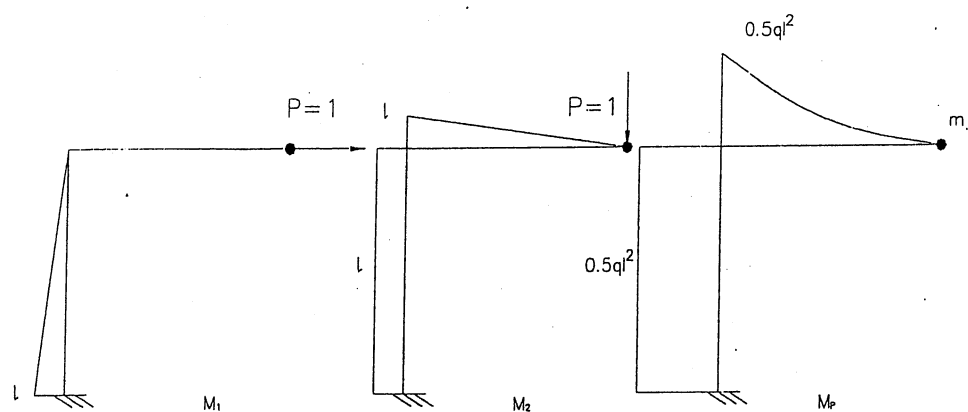
五、



$$\Delta_{KP} = \sum \int k \frac{\bar{Q}_K Q_P}{GA} ds + \sum \int \frac{\bar{M}_K M_P}{EI} ds = \frac{5ql^4}{24EI} + \frac{9ql^2}{5GA}$$

六、

该结构体系又两个自由度，水平方向和竖直方向



$$\delta_{11} = \frac{1}{EI} \frac{1}{2} l \times l \times \frac{2l}{3} = \frac{l^3}{3EI}, \delta_{11} = \frac{1}{EI} \left(\frac{1}{2} l \times l \times \frac{2l}{3} + l \times l \times l \right) = \frac{4l^3}{3EI}, \delta_{12} = \delta_{21} = \frac{1}{EI} \left(\frac{1}{2} l \times l \times l \right) = \frac{l^3}{2EI}$$

$$\Delta_{1P} = \frac{1}{EI} \frac{1}{2} l \times l \times \frac{ql^2}{2} = \frac{ql^4}{4EI}, \Delta_{2P} = \frac{1}{EI} \left(l \times l \times \frac{ql^2}{2} + \frac{1}{3} \times l \times \frac{ql^2}{2} \times \frac{3l}{4} \right) = \frac{5ql^4}{8EI}$$

代入 $Y + \delta M \ddot{Y} = \Delta_P \sin \theta t$

运动方程为

$$\left. \begin{aligned} y_1 + \frac{l^3}{3EI} m \ddot{y}_1 + \frac{l^3}{2EI} m \ddot{y}_2 &= \frac{ql^4}{4EI} \sin \theta t \\ y_2 + \frac{l^3}{2EI} m \ddot{y}_1 + \frac{4l^3}{3EI} m \ddot{y}_2 &= \frac{5ql^4}{8EI} \sin \theta t \end{aligned} \right\}$$

天大考 QQ:1264855369
研资料 15122905501

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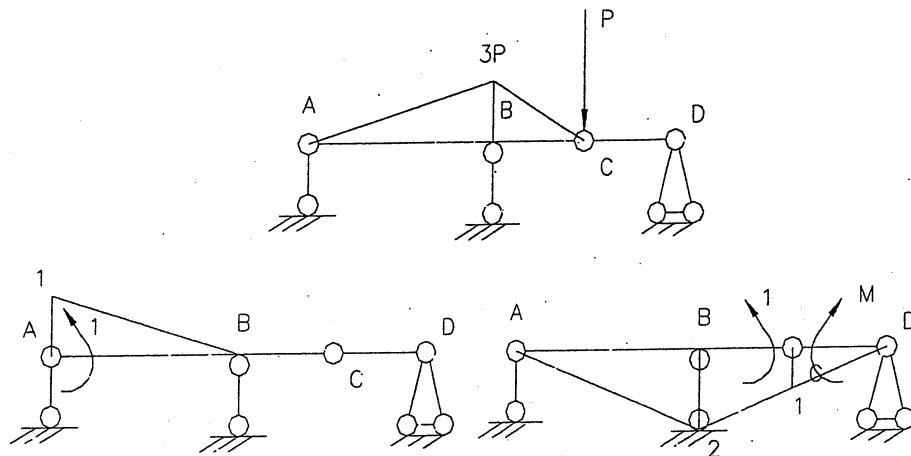
一、

1. 错误，该体系为几何不变体系，且有一个多余约束。将 ABF、CDG 及基础各看成一个刚片，有三个铰联结，且链杆 AD 为多于约束。
2. 错误，单位荷载法也可用于其它结构。

二、

1. 静定结构的计算用平衡条件计算即可，不涉及刚度。

2.



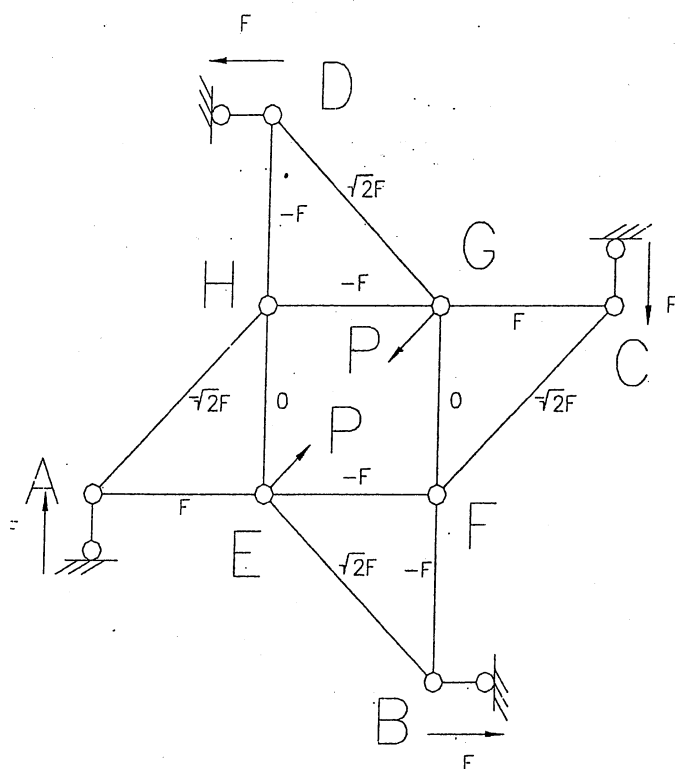
因 $\frac{1}{EI} \times \frac{1}{2} \times 6 \times 3P \times \frac{1}{3} = 0.001$ ，故 $-\frac{1}{EI} \times \frac{1}{2} \times 6 \times 3P \times \frac{4}{3} - \frac{3}{6EI} (2 \times 3P \times 2 + 3P \times 1) = -0.0065$ （方向与图示相反）。

3. $\omega = \sqrt{\frac{4k}{11m}}$ ，该体系有一个自由度 θ ，根据力矩平衡得 $11ma^2\ddot{\theta} + 4ka^2\theta = 0$ ，可得自振频率。

三、

利用水平方向和竖直方向的平衡，同时对中心取矩可得四个支座的力是大小相等的。

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天大考QQ:1264855369
研资料QQ:122905501

根据 G 点平衡可得 $F = \frac{\sqrt{2}}{2}P$ ，根据各点平衡得：

$$F = \frac{\sqrt{2}}{2}PF_{DG} = P, F_{HG} = -\frac{\sqrt{2}}{2}P, F_{HE} = 0$$

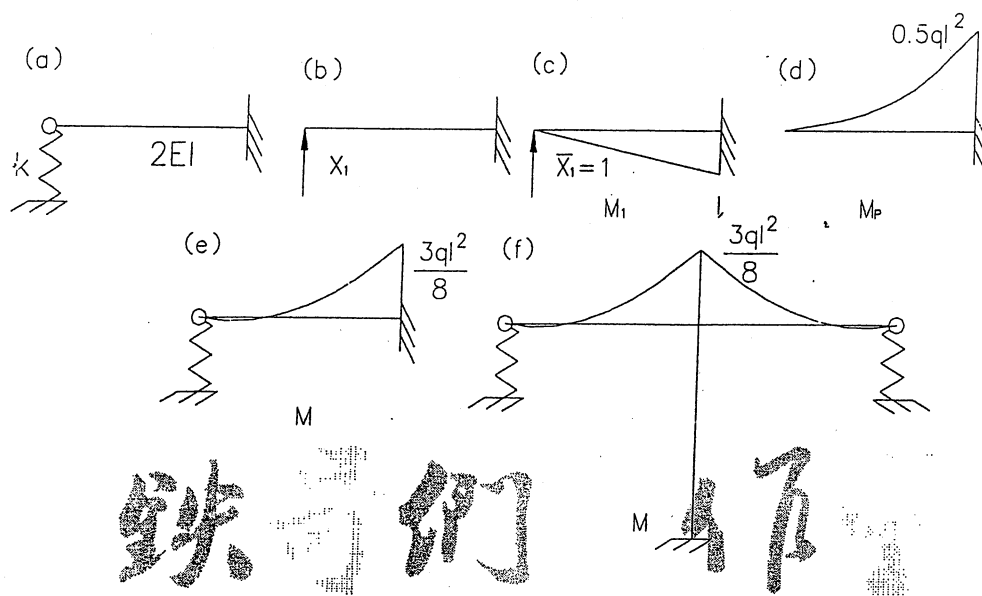
四、

结构荷载均对称，可取半结构进行计算，典型方程 $\delta_{11}X_1 + \Delta_{1P} = -\frac{X_1}{k}$

$$\delta_{11} = \frac{1}{2EI} \times \frac{1}{2} \times l \times l \times \frac{2l}{3} = \frac{l^3}{6EI}, \Delta_{1P} = -\frac{1}{2EI} \times \frac{1}{2} \times l \times \frac{1}{2}ql^2 \times \frac{3l}{4} = -\frac{ql^4}{16EI}$$

$$\text{得 } X_1 = \frac{ql}{8}$$

由 $M = \overline{M}_1X_1 + M_P$ 绘出弯矩图，再由结构体系的对称性绘出整体结构的弯矩图。



五、

典型方程

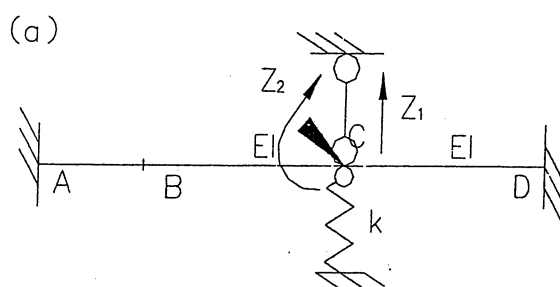
$$\begin{cases} r_{11}Z_1 + r_{12}Z_2 + R_{1\Delta} = 0 \\ r_{21}Z_1 + r_{22}Z_2 + R_{2\Delta} = 0 \end{cases}$$

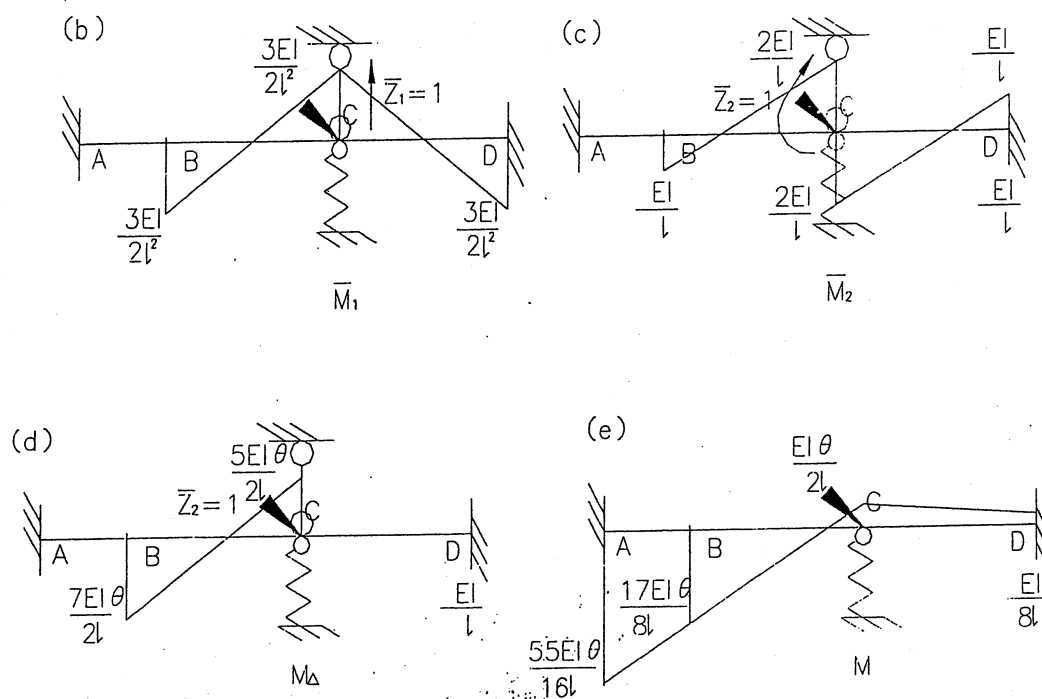
注意在 B 点既有角位移还有线位移，

$$r_{11} = \frac{3EI}{l^3} + k = \frac{6EI}{l^3}, r_{12} = r_{21} = 0, r_{22} = \frac{4EI}{l}, R_{1\Delta} = \frac{6EI\theta}{l}, R_{2\Delta} = \frac{5EI\theta}{2l}$$

$$Z_1 = -\frac{\theta l}{2}, Z_2 = -\frac{5\theta}{8}$$

利用 $M = \bar{M}_1Z_1 + \bar{M}_2Z_2 + M_{\Delta}$ 绘出结构的弯矩图，AB 段的弯矩由于 AC 段剪力未变，可直接由 BC 段弯矩图延伸过来。



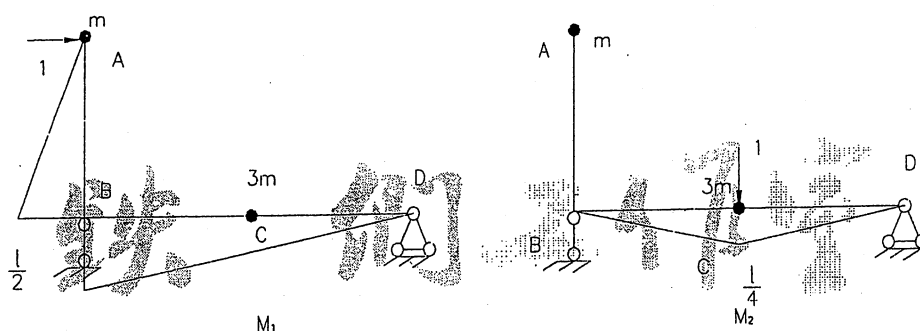


天大考QQ:126485569
研资料Tel:15122305501

六、
结构有两个自由度

$$\delta_{11} = \frac{1}{EI} \times \left(\frac{1}{2} \times \frac{l}{2} \times \frac{l}{2} \times \frac{l}{3} + \frac{1}{2} \times l \times \frac{l}{2} \times \frac{l}{3} \right) = \frac{l^3}{8EI}, \delta_{22} = \frac{2}{EI} \times \frac{1}{2} \times \frac{l}{2} \times \frac{l}{4} \times \frac{l}{6} = \frac{l^3}{48EI}$$

$$\delta_{12} = \delta_{21} = \frac{1}{EI} \times \frac{1}{2} \times l \times \frac{l}{4} \times \frac{l}{4} = \frac{l^3}{32EI}$$



$$\text{代入 } FM\ddot{Y} + Y = 0, \left| FM - \frac{1}{\omega^2} I \right| = 0$$

$$\text{设 } \lambda = \frac{ml^3}{32EI}, \begin{vmatrix} 4\lambda - \frac{1}{\omega^2} & 3 \\ 1 & 2\lambda - \frac{1}{\omega^2} \end{vmatrix} = 0 \text{ 得 } \omega_1 = \sqrt{\frac{1}{5\lambda}} = \sqrt{\frac{32EI}{5ml^3}}, \omega_2 = \sqrt{\frac{1}{\lambda}} = \sqrt{\frac{32EI}{ml^3}}$$

将 $\frac{1}{\omega^2} = \lambda, \frac{1}{\omega^2} = 5\lambda$ 分别代入 $\left(FM - \frac{1}{\omega_i^2}I\right)\Phi_i = 0$ 得

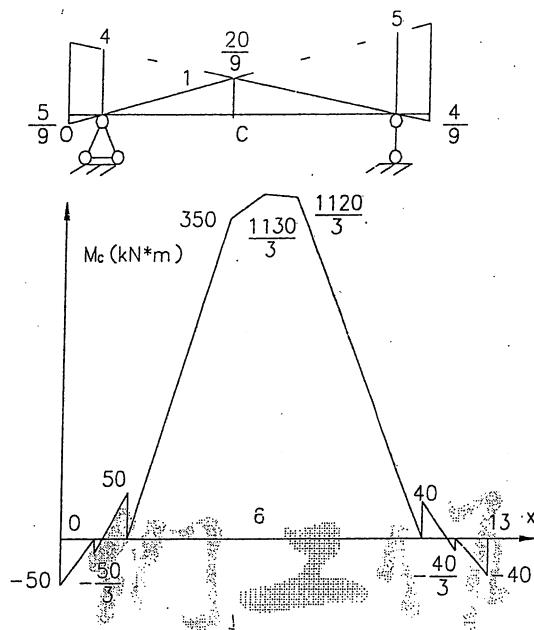
$$\begin{pmatrix} 3\lambda & 3 \\ 1 & \lambda \end{pmatrix} \begin{pmatrix} \varphi_1 \\ 1 \end{pmatrix} = 0, \begin{pmatrix} -\lambda & 3 \\ 1 & -3\lambda \end{pmatrix} \begin{pmatrix} \varphi_2 \\ 1 \end{pmatrix} = 0$$

得 $\varphi_1 = -1, \varphi_2 = 3$

振型 $\Phi_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}, \Phi_2 = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ 。

七、

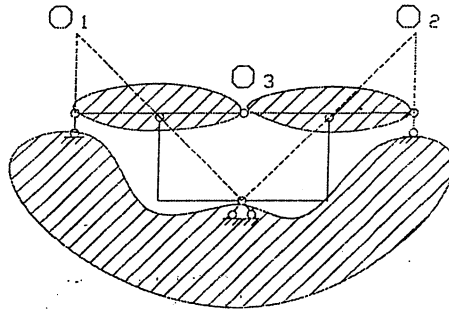
利用影响线做图，从荷载右端开始接触结构到左端离开结构，以右端荷载距离 0 点的距离为横轴，C 点的弯矩为纵轴，绘出 C 点弯矩有突变的点，然后用折线将点连接起来就为 C 点弯矩变化图，注意在涉及支座时，图形会有突变。



天津大学 2010 年结构力学真题参考答案

一、

1、几何不变体系且无多余约束。

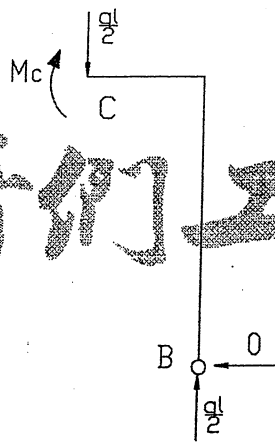


如图所示，根据三刚片规则，三铰不在一条直线上，可知该体系为几何不变体系且无多余约束。

2、 $\frac{ql^2}{4}$ 。(↗ ↖)

由三刚片规则知该结构为静定结构，且类似于三铰拱结构。分别取整体平衡和隔离体平衡即可得出结果。取整体平衡，对 A 点取矩得 B 支座竖向反力为 $ql/2$ (↑)；取右隔离体平衡，对 B 点取矩得：

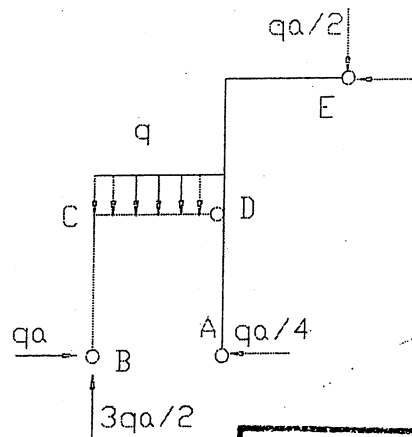
$$\frac{ql}{2} \cdot \frac{l}{2} = M_c$$



3、 $\frac{qa}{4}$ (←)。

由对称性可知 F_{By} 为 $3qa/2$ (↑)；取隔离体 BCD，对 D 点取矩得： $F_{Bx} = qa$ (→)；取隔离体 ABCDE，

对 E 点取矩得： $F_A = qa/4$ (←)

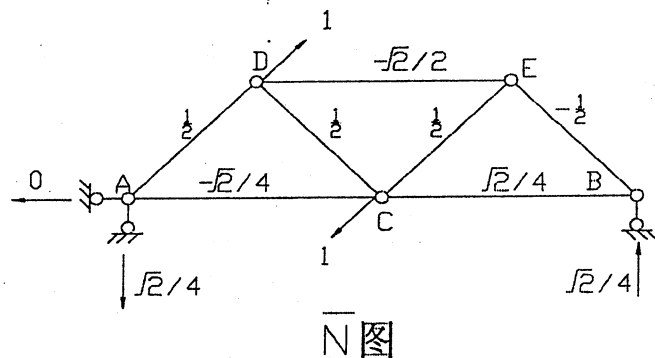
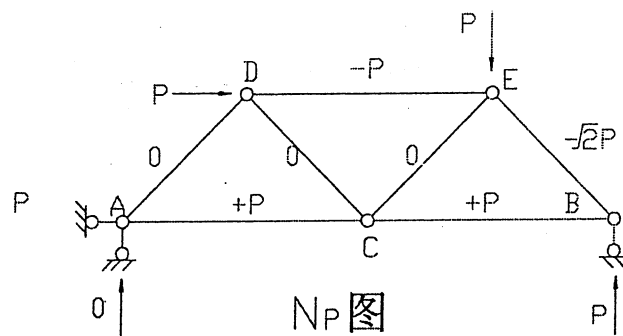


4、 $\frac{(2+\sqrt{2})}{2EA}P$ (顺时针)。

图乘得： $\theta = \frac{1}{EA} \left[(-P)(-\frac{\sqrt{2}}{2}) \cdot 2a + (-\sqrt{2}P)(-\frac{1}{2}) \cdot \sqrt{2}a \right] = \frac{(\sqrt{2}+1)Pa}{EA}$

相对转角为： $\frac{(\sqrt{2}+1)Pa}{EA} / \sqrt{2}a = \frac{(2+\sqrt{2})}{2EA}P$

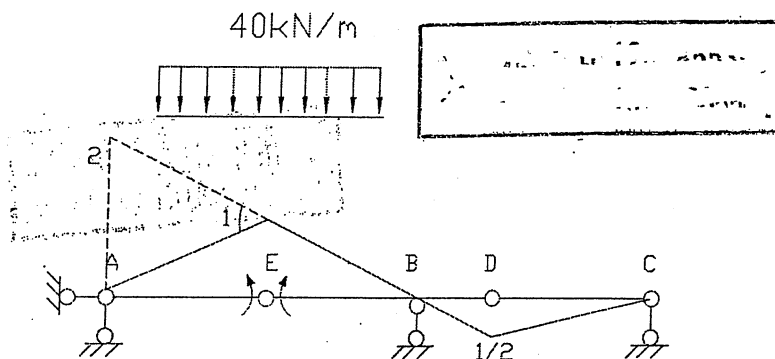
注：或者在 $\bar{N}$ 图中的杆两端施加一对大小为 $\frac{1}{\sqrt{2}a}$ 、方向相反的集中力，得出的结果即为相对转角。



5、75 kN·m (下侧受拉)。

根据机动法作出 E 截面的弯矩影响线，如下图所示。

$$M_{E_{\max}} = \left(\frac{1}{2} \cdot 4 \cdot 1 - \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{4} \cdot 2 \right) \cdot 40 = 75 \text{ kN} \cdot \text{m}$$



$$6、m\ddot{y} + \frac{3EI}{5l^3}y = \frac{3}{5}P(t)$$

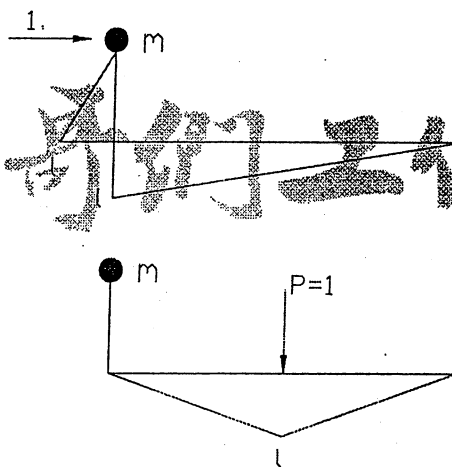
该题目为单自由度结构在简谐荷载作用下的强迫振动，且干扰力 $P(t)$ 不直接作用在质点上的情形。

设任意时刻质点 m 处的位移为 y ，则作用在质点 m 上的惯性力为 $F_I = -m\ddot{y}$ ，在惯性力 F_I 和干扰力 $P(t)$ 共同作用下，质点 m 处的位移为：

$$y = \delta_{11}F_I + \delta_{12}P(t) = \delta_{11}(-m\ddot{y}) + \delta_{12}P(t)$$

$$\text{将 } \delta_{11} = \frac{1}{EI} \left(\frac{1}{2} \cdot l \cdot l \cdot \frac{2l}{3} + \frac{1}{2} \cdot 4l \cdot l \cdot \frac{2l}{3} \right) = \frac{5l^3}{3EI}; \quad \delta_{12} = \frac{1}{EI} \cdot \frac{1}{2} \cdot 4l \cdot l \cdot \frac{l}{2} = \frac{l^3}{EI} \text{ 代入上式整理得:}$$

$$m\ddot{y} + \frac{3EI}{5l^3}y = \frac{3}{5}P(t)$$



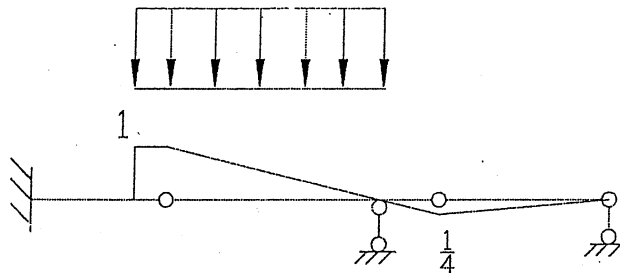
7、力法基本方程为：位移协调方程；或者 $\delta_{11}X_1 + \delta_{12}X_2 + \Delta_{1P} = 0$ (以结构具有两个未知量为例)
 $\delta_{21}X_1 + \delta_{22}X_2 + \Delta_{2P} = 0$

位移法基本方程为：平衡方程；或者 $r_{11}Z_1 + r_{12}Z_2 + R_{1P} = 0$ (以结构具有两个未知量为例)
 $r_{21}Z_1 + r_{22}Z_2 + R_{2P} = 0$

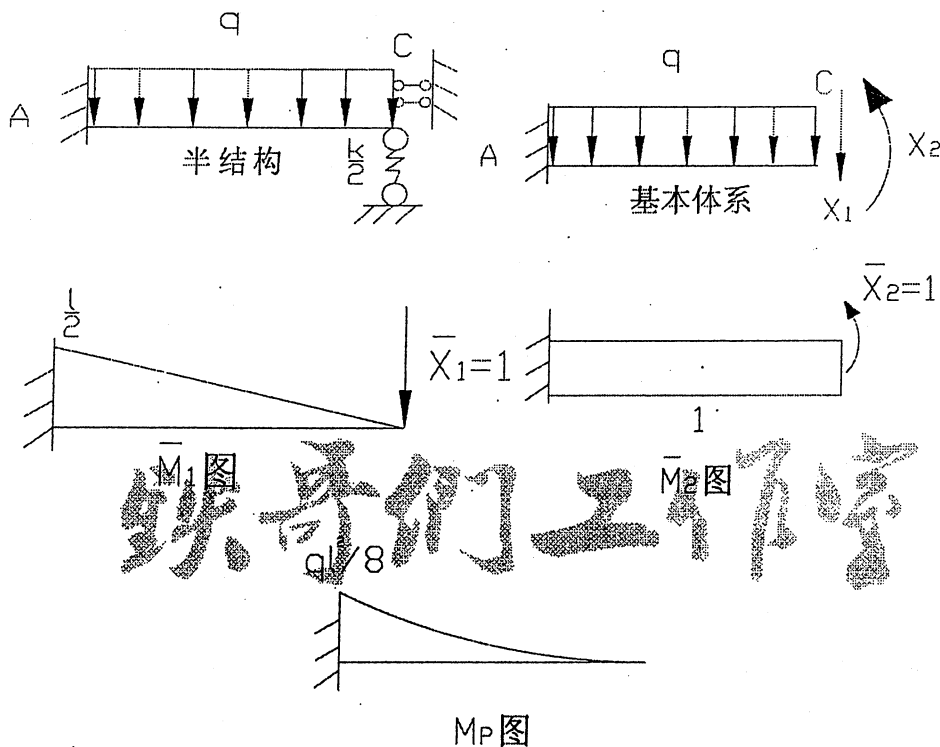
8、100kN。

根据机动法，作出K截面剪力影响线，如下图所示。

$$Q_{K\max} = (1 \cdot 1 + \frac{1}{2} \cdot 8 \cdot 1) \times 20 = 100 \text{ kN}$$



二、忽略轴向变形。半结构及基本体系如下图所示。



$$\left. \begin{aligned} \delta_{11}X_1 + \delta_{12}X_2 + \Delta_{1P} &= -\frac{2X_1}{k} \\ \delta_{21}X_1 + \delta_{22}X_2 + \Delta_{2P} &= 0 \end{aligned} \right\}$$

力法典型方程

$$\delta_{11} = \frac{1}{EI} \times \frac{1}{2} \times \frac{l}{2} \times \frac{l}{2} \times \frac{2}{3} = \frac{l^3}{24EI}, \quad \delta_{22} = \frac{1}{EI} \times \frac{l}{2} \times 1 \times 1 = \frac{l}{2EI}$$

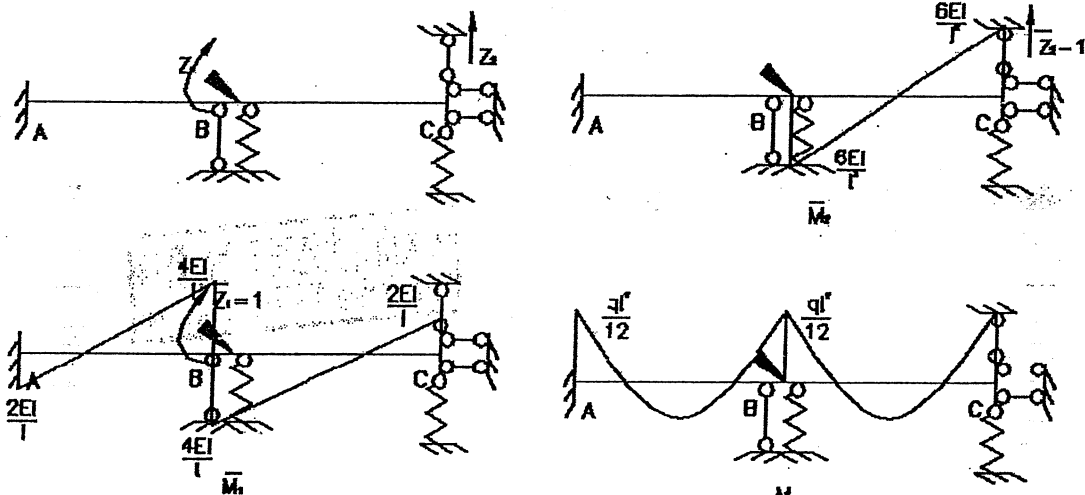
$$\delta_{12} = \delta_{21} = -\frac{1}{EI} \times \frac{1}{2} \times \frac{l}{2} \times \frac{l}{2} \times 1 = -\frac{l^2}{8EI}$$

$$\Delta_{1P} = \frac{1}{EI} \left(\frac{1}{3} \times \frac{l}{2} \times \frac{ql^2}{8} \times \frac{3}{4} \times \frac{l}{2} \right) = \frac{ql^4}{128EI}, \quad \Delta_{2P} = -\frac{1}{EI} \left(\frac{1}{3} \times \frac{l}{2} \times \frac{ql^2}{8} \times 1 \right) = -\frac{ql^3}{48EI}$$

由上图可知，AB梁中点C处截面弯矩为0，有 $M_C = 1 \times X_2 = 0, X_2 = 0$

将 $X_2 = 0$ 代入典型方程解得 $k = \frac{384EI}{l^3}$ 。

三、

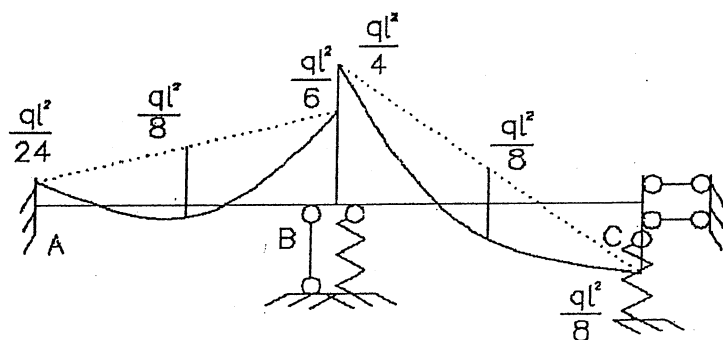


典型方程 $\begin{cases} r_{11}Z_1 + r_{12}Z_2 + R_{1P} = 0 \\ r_{21}Z_1 + r_{22}Z_2 + R_{2P} = 0 \end{cases}$

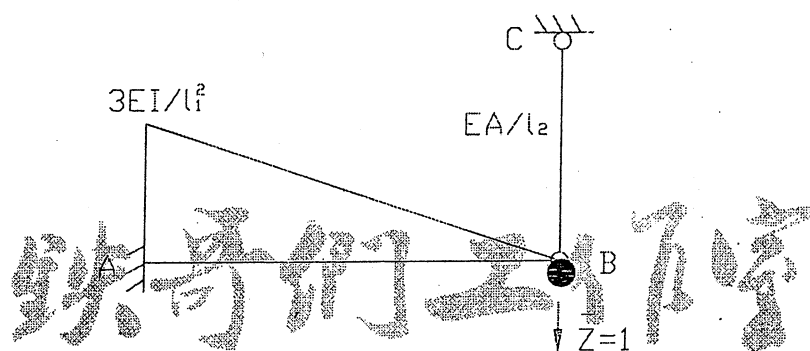
$$r_{11} = \frac{12EI}{l}, r_{12} = r_{21} = \frac{6EI}{l^2}, r_{22} = \frac{12EI}{l^3} + \frac{3EI}{l^3} = \frac{15EI}{l^3}, R_{1P} = 0, R_{2P} = \frac{ql}{2}$$

$$Z_1 = \frac{ql^3}{48EI}, Z_2 = -\frac{ql^4}{24EI}$$

由 $M = \overline{M}_1 Z_1 + \overline{M}_2 Z_2 + M_P$ 绘出弯矩图。



四、



(1) 由刚度的物理意义得，

$$k = \frac{3EI}{l_1^3} + \frac{EA}{l_2}$$

$$k = 51.25 \times 10^3 \text{ N/m}$$

$$m = \frac{G}{g} = \frac{10 \times 10^3}{10} \text{ kg} = 10^3 \text{ kg}$$

$$\omega = \sqrt{\frac{k}{m}} = 7.2 \frac{1}{s}$$

(2) 结构为自由振动

$$A = \sqrt{y_0^2 + (\dot{y}/\omega)^2} = \sqrt{0.01^2 + 0.04^2/7.2^2} = 0.011 \text{ m}$$

五、

结构具有两个自由度，利用柔度矩阵的物理意义求柔度矩阵。

$$\delta_{12} = \delta_{21} = 0$$

$$FM\ddot{Y} + Y = 0, \left| FM - \frac{1}{\omega^2} I \right| = 0$$

鉄哥们..



