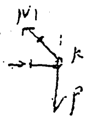
天大历年真题答案

$\boxed{一九九四}$

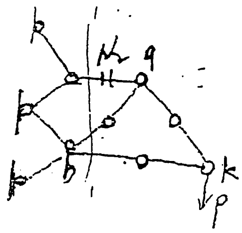
一、解：因FJ, IJ, ~~GF, ME, HM~~ 为零杆

对K结点受力分析



$\frac{\sqrt{2}}{2}N_1 = P$　$N_1=\sqrt{2}P$（拉力）

因 IG, IM, HM, EH, EF 用析为零杆，结构可简化



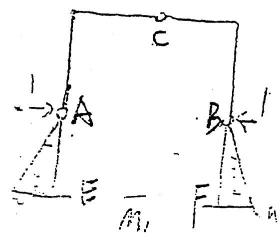
截取H截面右部分析

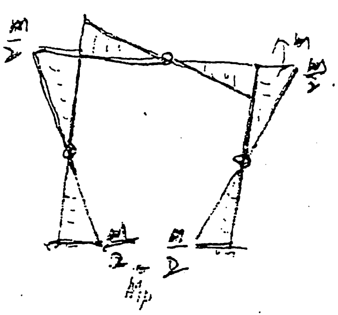
$\sum M_b = 0$

$\Rightarrow N_2\cdot 2a = P\cdot 4a$

$\Rightarrow N_2 = \frac{1}{2}P$

二、解：相对转角 $\Delta = \Delta_p + \Delta_t$





① $\Delta_p = \int \frac{\bar{M}_1 M_p}{EI} ds = \frac{-1}{EI}\left(\frac{1}{2}a\times a\cdot\frac{2}{3}\cdot\frac{M}{2}\right)\times 2$

$= \frac{-Ma^2}{3EI}$（与假设方向相反）

② $\Delta t = t$

$t_0 = \frac{t}{2}$　仅在AC部分温度升高t°C

$\Delta_t = \sum\alpha t_0\int \bar{N} ds + \sum\frac{\alpha\Delta t}{h}\int \bar{M} ds = 0$

~~$= 0 + \frac{\alpha\cdot t}{2h}\cdot\frac{1}{2}a^2 = \frac{\alpha\cdot ta^2}{4h}$~~

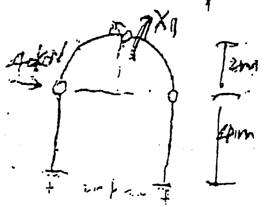
$\therefore \Delta = \frac{\alpha t a^2}{4h} - \frac{Ma^2}{3EI}$

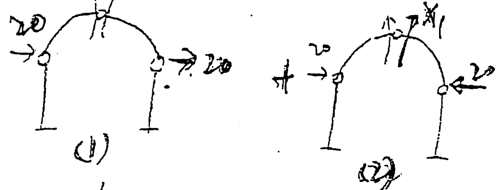
三、解：1 先计算自由度 $W = 3m - 2n - C_0 - C$

取 $m=3$　$n=2$，$C_0=0$，$C=6$

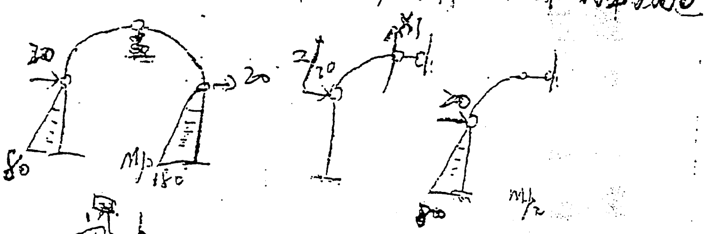
$\therefore W = 9-4-6 = -1$　即一次超静定

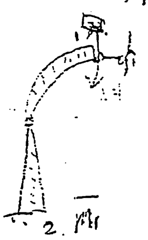
基本体系为：





(1) 根据反对称荷载作用在对称结构中，对称未知力为零





$\delta_{11}X_1 + \Delta_p = 0$

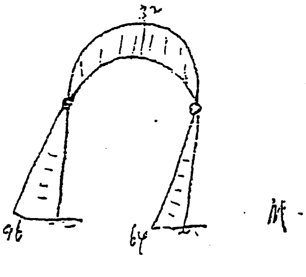
$M = \bar{M}_1 X_1 + M_{p2} + M_{p1}$

$\Delta_p = -\frac{1}{EI}\left(\frac{1}{2}\times 80\times 4\times\frac{2}{3}\times 2\right) = \frac{-160}{3EI}$

$\delta_{11} = \int\frac{\bar{M}_1^2}{EI}ds = \frac{1}{EI}\int_0^{\frac{\pi}{2}}\left(\frac{1}{2}R\sin\theta\right)^2 R\,d\theta +$

$= \frac{8}{EI}\left(\frac{\pi}{2}-\frac{1}{2}\right) = \frac{\pi-1}{2EI} = \frac{5}{EI}$

$\Rightarrow X_1 = 32$



天大历年真题答案集

解：取B、C、E处铰水平位移△为未知量，

$M_{AB} = -\frac{1}{8}ql^2 - \frac{3i_{AB}}{l}\triangle = -20 - 3\frac{EI}{16}\triangle$

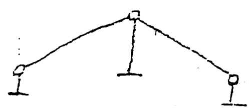
$M_{BA} = 0$， $F_{QBA} = \frac{\frac{3EI}{16}\triangle}{4}$

同 $M_{DC} = -\frac{3i_{DC}}{l_{DC}}\triangle = \frac{-3EI}{6\times 6}\triangle = -\frac{EI}{12}\triangle$

$F_{QCD} = \frac{EI\triangle}{72}$

$M_{FE} = \frac{-3i_{EF}}{l}\triangle = \frac{-3EI}{16}\triangle$

$F_{QEF} = \frac{3EI}{64}\triangle$



$\Sigma F_x = 0$

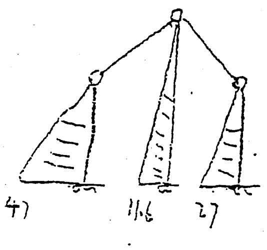
$\frac{3}{64}EI\triangle - 11 + \frac{1}{72}EI\triangle + \frac{3}{64}EI\triangle = 0$

$\Rightarrow EI\triangle = \frac{32\times 11\times 9}{31} =$

$\therefore M_{AB} = -20 - 3\frac{1}{16}\times\frac{32\times 11\times 9}{31} = -47$

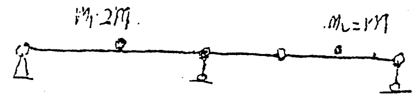
$M_{DC} = -\frac{1}{12}\cdot\frac{32\times 11\times 9}{31} = -11.6$

$M_{FE} = -\frac{3}{16}EI\triangle = -27$



M图

题六：解：



两个自由度的自由振动

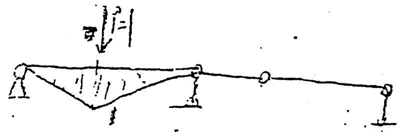
由计算式

$(m_1\delta_{11} - \frac{1}{W^2})Y_1 + m_2\delta_{12}Y_2 = 0$

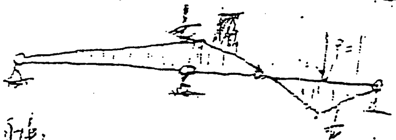
$m_1\delta_{21}Y_1 + (m_2\delta_{22} - \frac{1}{W^2})Y_2 = 0$

设 $\lambda = \frac{1}{w^2}$

$$\lambda_{\frac{1}{2}} = \frac{(m_1\delta_{11}+m_2\delta_{22}) \pm\sqrt{(m_1\delta_{11}+m_2\delta_{22})^2 - 4m_1m_2(\delta_{11}\delta_{22}-\delta_{12}\delta_{21})}}{2}$$



$\bar{M}_1$图



$\bar{M}_2$图

图乘得：

$\delta_{11} = \frac{1}{EI}\cdot(\frac{1}{2}\times 2\times 1\times\frac{2}{3}\times 1\times 2) = \frac{4}{3EI}$

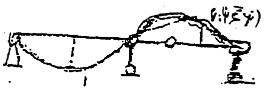
$\delta_{22} = \frac{1}{EI}(\frac{1}{2}\times 1\times\frac{1}{2}\times\frac{2}{3}\times\frac{1}{2}\times 3 + \frac{1}{2}\times 4\times\frac{1}{2}\times\frac{2}{3}\times\frac{1}{2}) = \frac{7}{12EI}$

$\delta_{12} = \frac{1}{EI}(\frac{1}{2}\times 4\times 1\times\frac{1}{4}) = \frac{-1}{2EI}$

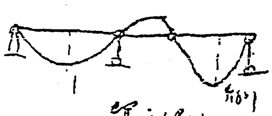
故 $\lambda_{\frac{1}{2}} = \frac{m}{EI}(1.625 \pm 1.2149) \Rightarrow \lambda_1 = 2.844\frac{m}{EI} \Rightarrow W_1 = 0.593\sqrt{\frac{EI}{m}}$

$\lambda_2 = 0.316\frac{m}{EI} \Rightarrow W_2 = 1.613\sqrt{\frac{EI}{m}}$

$\frac{Y_{11}}{Y_{12}} = \frac{-m_2\delta_{12}}{m_1\delta_{11} - \frac{1}{W_1^2}} = \frac{-1}{0.4347}$　　$\frac{Y_{21}}{Y_{22}} = \frac{-m_2\delta_{12}}{m_1\delta_{11} - \frac{1}{W_2^2}} = \frac{1}{4.601}$



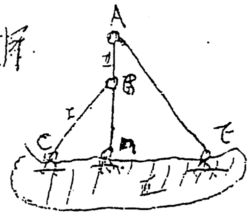
第一振型



第二振型

九九五 [95年]

一、选择



$m=4, h=3, C=6$

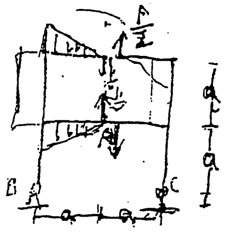
$W=3m-2h-C=12-6-6=0$

且无多余约束

取AD杆为刚片III，BC为I，地基为II

I、II 铰B，I与III 铰C，II与III 虚铰A

ABC不共线 故几何不变　[A]



BC支座均无水平反力

故上端 $\frac{P}{2}$ 力偶矩在A截面抵消　$\therefore A_{水}=\frac{P}{2}$

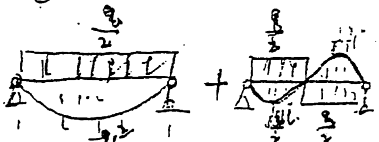
[D]

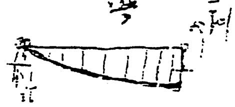
由图示 1~8 杆均为零杆　[C]

[A]　$3m-2n-C_0-C$

用静定结构的位移计算

用超静定结构的位移计算

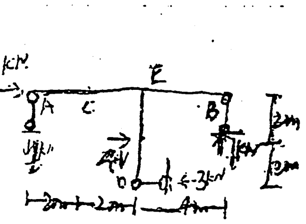


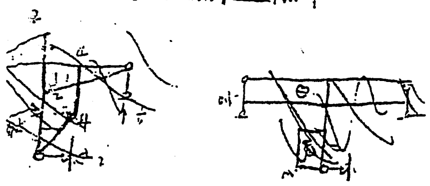


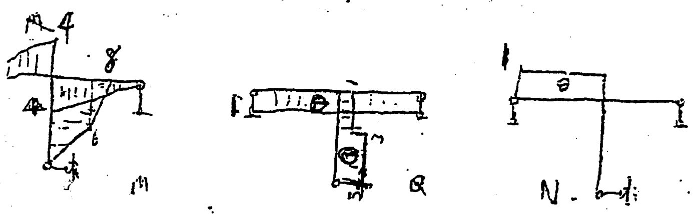
叠加

$$\Delta=\Delta_1+\Delta_2=\frac{1}{EI}\left(\frac{2}{3}\times 2l\times\frac{1}{4}ql\times\frac{l}{2}\right)+\frac{1}{EI}\left[\frac{2}{3}\times\cdots\right]=\frac{7ql^3}{48EI}(\uparrow)$$

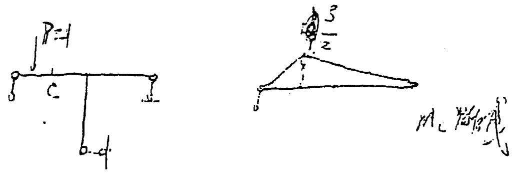
二、计 1)



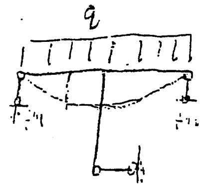




二、计 2)

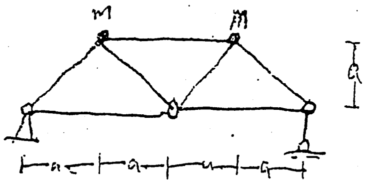


M_C 影响线



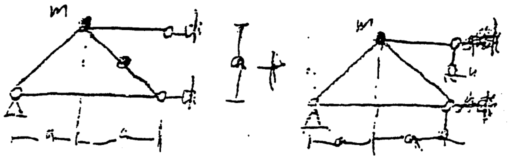
$M_C=\frac{1}{2}\times q\times 8\times 2-q\times 2\times 1=6q$

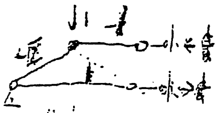
三、计



因不计水平运动，故只有竖向两个自由度

因结构对称，质量也对称，可取半结构计算

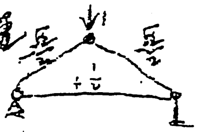




$\delta_1=\frac{\sum\bar{N}_1^2 l}{EA}=\frac{1}{EA}\left(\cdots+\frac{1}{\cdots}\cdot 2a+\frac{2}{\cdots}\times\sqrt{2}a\right)=\frac{4.4a}{EA}$

$W_1=\sqrt{\frac{1}{m\delta_1}}=\sqrt{\frac{EA}{4.4ma}}$

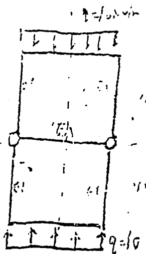
反对称



$\delta_2=\frac{\sum\bar{N}^2 l}{EA}=\frac{1}{EA}\left(\frac{1}{2}\cdot\sqrt{2}a\times 2+\frac{1}{4}\times 2a\right)=\frac{1.9}{EA}a$

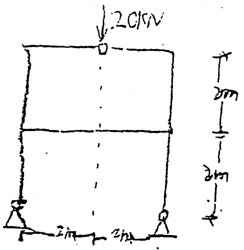
$W_2=\sqrt{\frac{1}{m\delta_2}}=\sqrt{\frac{EA}{1.9ma}}$

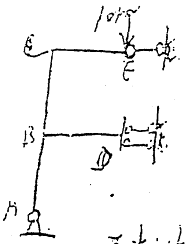
5.1:



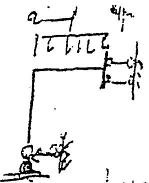
本结构有水平与竖向两对称轴，可取¼结构，利用对称荷载法

五.解:



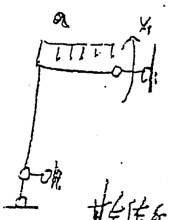


取半边结构



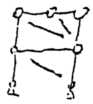
¼结构

一次超静定



基本体系

原结构4个角位移，3个线位移



半边结构有两个角位移 θ_B, θ_C

$M_{AB}=0$ $\quad M_{BA}=3i_{BA}\theta_B$, $\quad M_{BD}=i_{BD}\theta_B$ $\quad M_{DB}=-i_{BD}\theta_B$

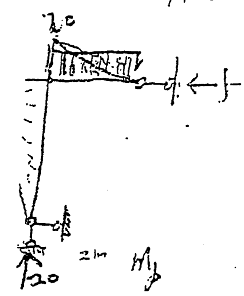
$M_{CB}=2i_{CB}\theta_B+4i_{CB}\theta_C$ $\quad M_{CE}=20$ $\quad M_{EC}=0$

$M_{BC}=2i\theta_C$

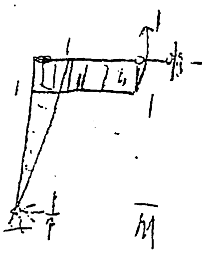
(取 $i=\frac{EI}{4}$, ~~$i_{BC}=2i$~~ ...)

~~$\Sigma M_B = 3i \cdot 2i\theta_B + i\theta_B + 2 \cdot 2i\theta_C = 0$~~

~~$\Sigma M_C = 2 \cdot 2i\theta_B + 4 \cdot 2i\theta_C - 20 = 0$~~



Mp



$\bar{M}$

$$\delta_{11}=\frac{1}{EI}\left(\frac{1}{2}\times4\times1\times\frac{2}{3}\times1+1\times2\times1\right)=\frac{10}{3EI}$$

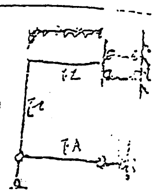
$$\Delta_{1P}=-\frac{1}{EI}\left(\frac{1}{2}\times4\times1\times\frac{2}{3}\times20+\frac{1}{3}\times2\times20\times1\right)=-\frac{40}{EI}\ ?$$

$$\begin{cases}\Sigma M_B = 3i\theta_B + i\theta_B + 4i\theta_B + 2i\theta_C = 0\\ \Sigma M_C = 2i\theta_B + 4i\theta_C - 20 = 0\end{cases}$$

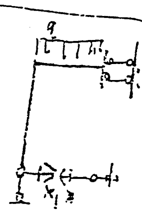
$$\Rightarrow \begin{cases}\theta_B = -\frac{10}{7i}\\ \theta_C = \frac{40}{7i}\end{cases}$$

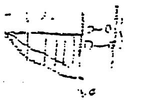
解:

基本体系结构

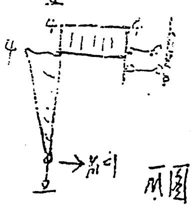


取基本体系





Mp图



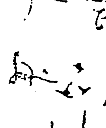
$\bar{M}$图 $X_1=1$

$$\delta_{11}=\frac{1}{EI}\left(\frac{1}{3}4^3+4\times2\times4\right)=\frac{160}{3EI}$$

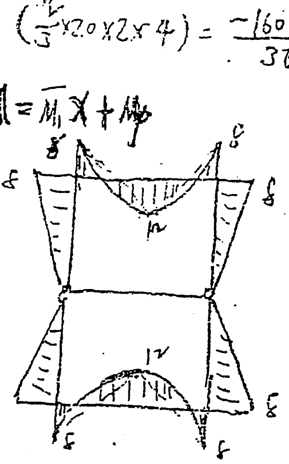
$$\Delta_{1P}=\frac{-1}{EI}\left(\frac{2}{3}\times20\times2\times4\right)=\frac{-160\times2}{3EI}$$

$$X_1=-\frac{\Delta_{1P}}{\delta_{11}}=2$$

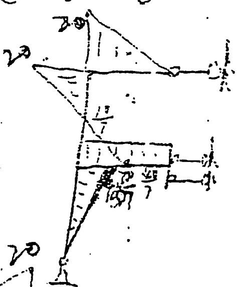
由 $M=\bar{M}_1X_1+M_P$

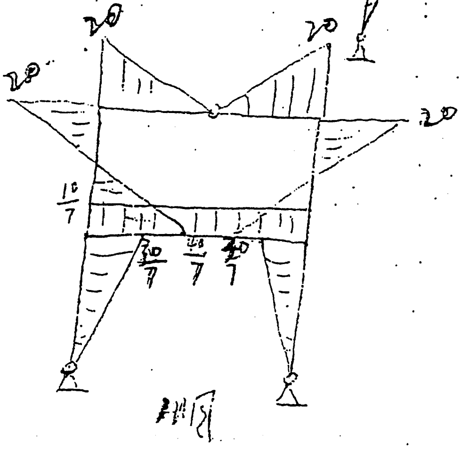


M图



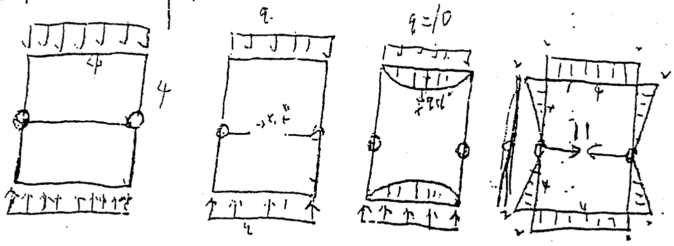
轴力易求.





M图

第四题解:



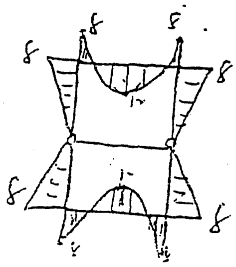
M_P $\quad$ $\overline{M}$

$$\delta_{11}=\frac{1}{EI}\left(2\times4\times2\times2+\frac{1}{2}\times4\times2\times\frac{2}{3}\times2\times4\right)+\frac{1^2\cdot4}{EA}=\frac{806}{15EI}$$

$$\Delta_{1P}=-\frac{1}{EI}\left(\frac{2}{3}\cdot4\times\frac{1}{8}\times10\times4\times4\times2\times2\right)=-\frac{640}{3EI}$$

$$X_1=-\frac{\Delta_{1P}}{\delta_{11}}=\frac{\frac{640}{3EI}}{\frac{806}{15EI}}\doteq4$$

$M=\overline{M}_1X_1+M_P$

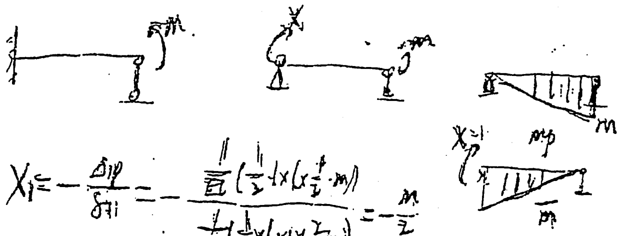


M图

$\boxed{\text{一九九六}}$

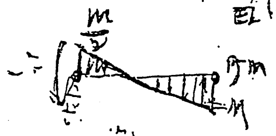
1. ✓ 2. ? 3. X

1.



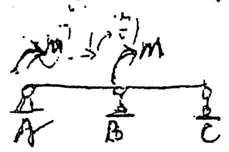
解 $X_1 = -\frac{\Delta_{1P}}{\delta_{11}} = -\frac{\frac{1}{EI}(\frac{1}{2} \cdot l \times (l \times \frac{1}{3} \cdot m))}{\frac{1}{EI}(\frac{1}{2} \times l \times l \times \frac{2}{3} l)} = -\frac{m}{2}$

∴

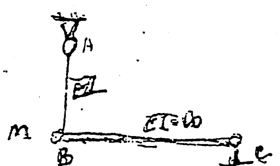


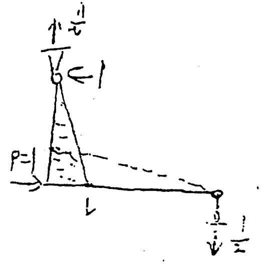
(B)

∴



[D]



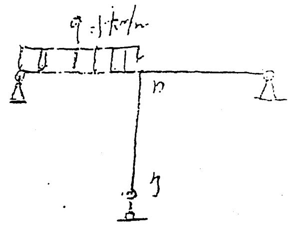


$\delta_{11} = \frac{1}{EI} \cdot \frac{1}{2} \times l \times l \times \frac{2}{3} \cdot l + 0 = \frac{l^3}{3EI}$

$\omega = \sqrt{\frac{1}{m\delta_{11}}} = \sqrt{\frac{3EI}{ml^3}}$

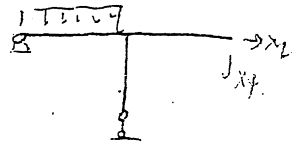
[A]

3.



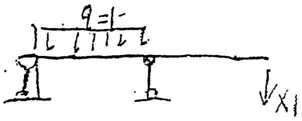
解: 结构为两次超静定

基本体系

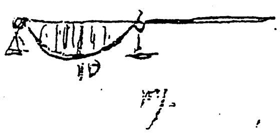


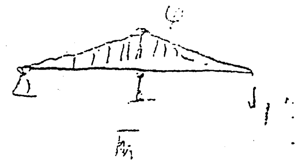
因 X_2 为 0, 因竖杆不承受水平作用力

故可将结构简化为



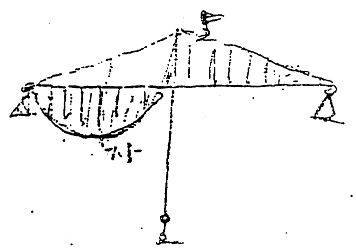
计算较...





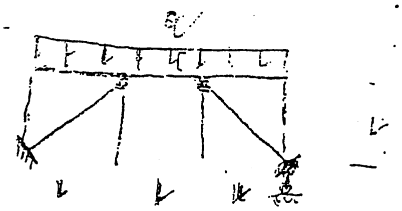
$X_1 = -\frac{\Delta_{1P}}{\delta_{11}} = -\frac{\frac{1}{EI}(\frac{2}{3} \times 4 \times 1 \times 2)}{\frac{1}{EI}(\frac{1}{2} \times 4 \times 4 \times \frac{2}{3} \times 4 \times 2)} = -\frac{5}{4}$

∴ $M = \bar{M}_1 X_1 + M_P$

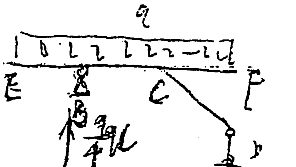


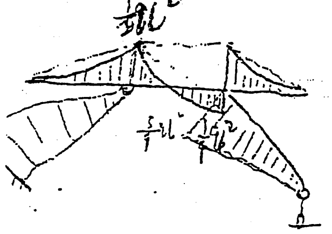
M图

1. 解:

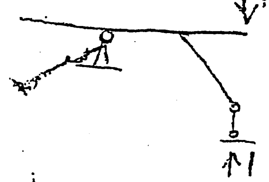


图是附属结构





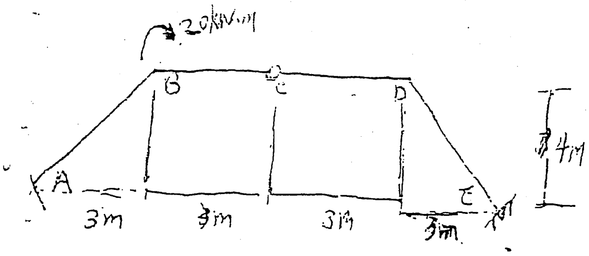
解

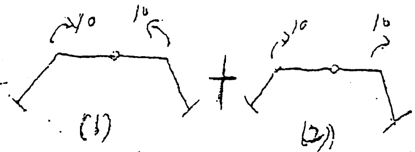


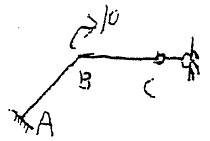
$\Delta_F = -\bar{F}_R \cdot C$

$= -(-1) \cdot 0.01$

$= 0.01\ m$



对称结构作分解 

1)对称分析 取半边结构 

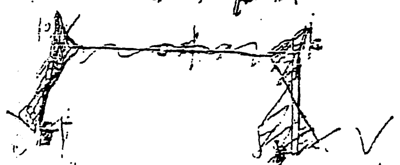
基本未知量 转角 θ_B.

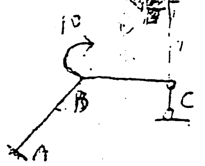
$M_{AB}=2i\theta_B=\frac{2EI}{5}\theta_B$ $M_{BA}=4i\theta_B=\frac{4EI}{5}\theta_B$

$M_{BC}=M_{CB}=0$. $\Sigma M_B=0$. $\frac{4}{5}EI\theta_B=10$.

$\Rightarrow EI\theta_B=\frac{25}{2}$ $M_{AB}=5$ kN·m

$M_{BA}=10$ kN·m.



2)反对称分析: 

基本未知量 θ_B, Δ_{BC}

$M_{AB}=2i\theta_B-\frac{6i}{l}\cdot\frac{5}{4}\Delta_{BC}$ 即 $=\frac{2EI}{5}\theta_B-\frac{3EI}{10}\Delta_{BC}$

$M_{BA}=4i\theta_B-\frac{6i}{l}\cdot\frac{5}{4}\Delta_{BC}$ 即 $=\frac{4EI}{5}\theta_B-\frac{3}{10}EI\Delta_{BC}$

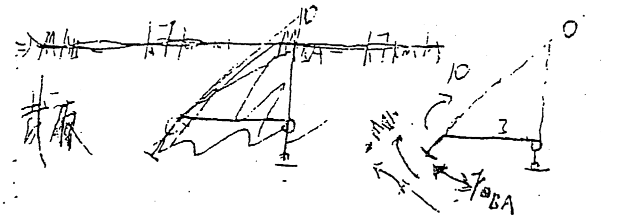
$M_{BC}=3i\theta_B-\frac{3i}{l}\cdot\frac{3}{4}\Delta_{BC}=\frac{EI\Delta_{BC}}{4}+EI\theta_B$

$M_{BC}=10 \Rightarrow \frac{9}{5}EI\theta_B-\frac{1}{20}EI\Delta_{BC}=10$ ①

~~取 $\Sigma F_x=0$~~

~~$\Rightarrow F_{QBA}=0$ 即 $\frac{M_{AB}+M_{BA}}{5}=0$~~

~~$\frac{6}{5}EI\theta_B-\frac{3}{5}EI\Delta_{BC}=0$ ② ①② $EI\theta_B=$, $EI\Delta_{BC}=\frac{50}{3}$~~

截取 

$\Sigma M_c=0$ $F_{QBA}\cdot 5+10+M_{BA}=0$

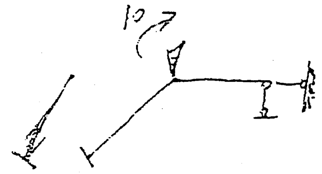
即 $\frac{M_{AB}-M_{BA}}{5}\cdot 5+10+M_{BA}=0$?

即 $M_{AB}-2M_{BA}+10=0$ ②

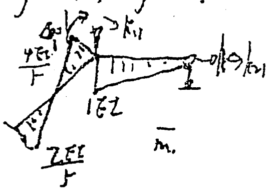
①② $\Rightarrow$ $EI\theta_B=$
$EI\Delta_{BC}=$?

在对O取矩时 不知 F_{QBA}、M_{BA} 的正负方向，故此结果错误！！而用结构位移法作时不产生歧义，不能出错！

位移法并(2)
用位移法求
总结

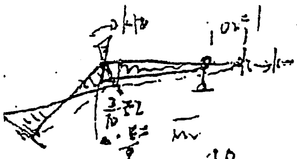


$F_{1P}=-10,\ F_{2P}=0.$

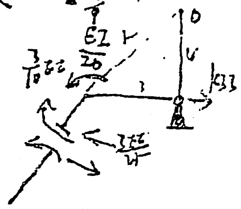


$k_{11}=\frac{4}{5}EI+1\cdot EI=\frac{9}{5}EI$

k_{21} 由 k_{12} 求



$k_{12}=\frac{-3}{10}EI+\frac{EI}{4}=-\frac{1}{20}EI$



$\Sigma M_0=0.\ k_{33}.$

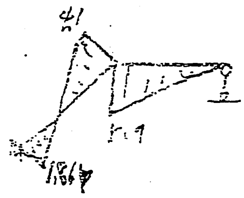
$k_{33}\cdot 4=\frac{-3}{4}EI\cdot 5+\frac{3}{10}EI.$

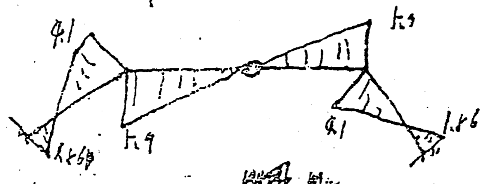
$k_{33}=\frac{9}{40}EI$ ×.

$k_{11}\Delta_1+k_{12}\Delta_2+F_{1P}=0$

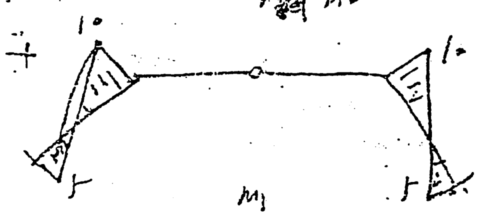
$k_{21}\Delta_1+k_{22}\Delta_2+F_{2P}=0$ $\Rightarrow\begin{cases}\Delta_1=5.59\\ \Delta_2=1.24\end{cases}$

$M=\overline{M}_1\Delta_1+\overline{M}_2\Delta_2$

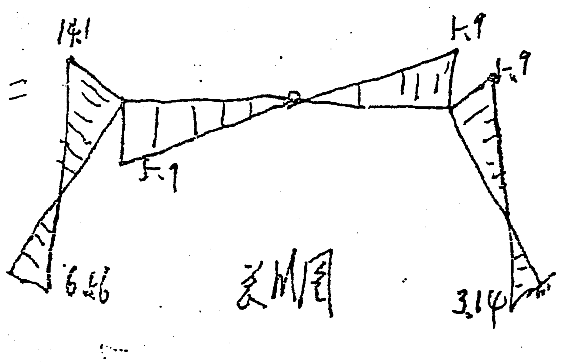




$+$

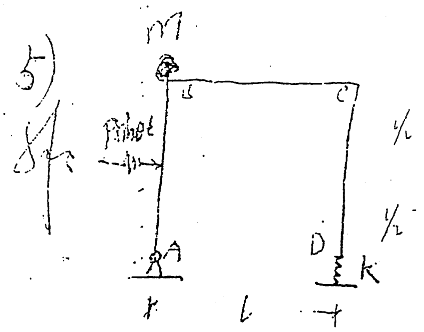


$=$

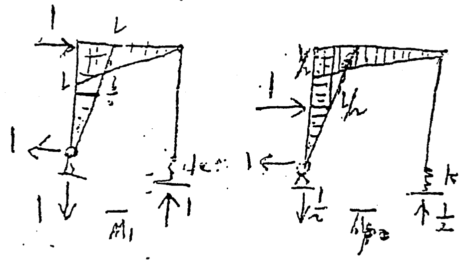


总M图

5)



单自由度：



$\delta_{11}=\frac{1}{EI}\cdot\left(\frac{1}{2}\times l\times l\times\frac{2}{3}l\times 2\right)-\Sigma\overline{F}_R C$

$=\frac{2l^3}{3EI}+1\cdot\frac{1}{k}=\frac{2l^3}{3EI}+\frac{2l^3}{EI}=\frac{8l^3}{3EI}$

$\delta_{1P}=\frac{1}{EI}\left(\frac{1}{2}\times l\times l\times\frac{2}{3}\times\frac{l}{2}+\frac{1}{2}\times\frac{l}{2}\times\frac{l}{2}\times\frac{2}{3}\times\frac{l}{2}+\frac{l}{2}\times\frac{l}{2}\times\frac{3}{4}l\right)-\Sigma\overline{F}_R\cdot C$

$=\frac{19l^3}{48EI}+\frac{1}{2}\cdot\frac{1}{k}=\frac{67l^3}{48EI}$

$W=\sqrt{\frac{1}{m\delta_{11}}}=\sqrt{\frac{3EI}{8ml^3}}$

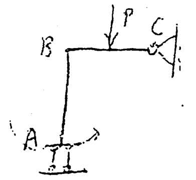
$\beta=\frac{1}{1-\frac{\theta^2}{\omega^2}}=\frac{1}{1-\frac{1}{4}}=\frac{4}{3}$

$y_{max}=\beta y_{st}=\beta\cdot P\cdot\delta_{1P}=\frac{4}{3}\times P\times\frac{67l^3}{48EI}$

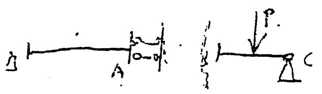
$=\frac{67Pl^3}{36EI}$

一九九七

1.



求M_{AB}：
用位移法求解，θ_B为基本未知量。



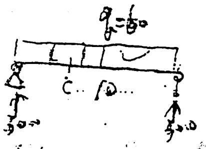
$M_{BA} = i\theta_B$，$M_{AB} = -i\theta_B$

$M_{BC} = 3i\theta_B - \frac{3}{16}Pl$　由 $\Sigma M_B = 0 \Rightarrow \theta_B = \frac{3Pl}{64i}$

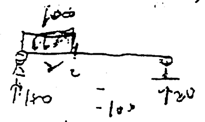
$\therefore M_{AB} = -\frac{3}{64}Pl$　左侧受拉

$\mu = \frac{1}{1-\left(\frac{\theta}{\omega}\right)^2}$　$\theta \le \omega$　$\frac{\theta}{\omega} \le 1$　$\Rightarrow 0 \le \left(\frac{\theta}{\omega}\right)^2 \le 1$

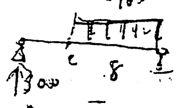
$0 \le 1-\left(\frac{\theta}{\omega}\right)^2 \le 1$　$\therefore \mu \in [1, +\infty)$



$V_c = 180\downarrow$



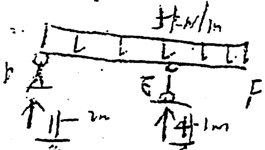
$V_D' = 20\uparrow$　$\therefore V_{max} = 160\downarrow$

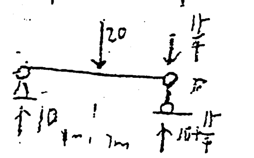


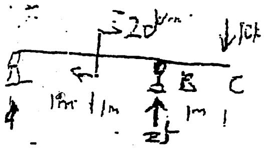
$V_D'' = 50\downarrow$　$V_{max} = 130\downarrow$

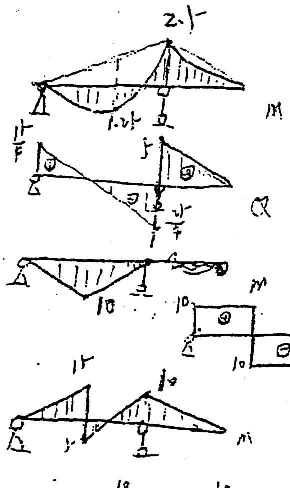
1.[C]　2. ?　3. D.

4. 静定多跨梁，依次取附属结构计算。

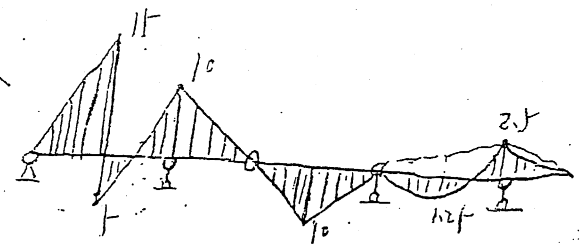




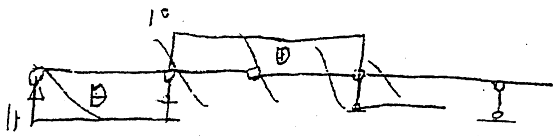


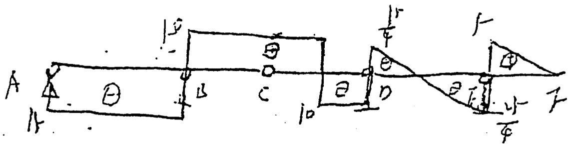


5. 故



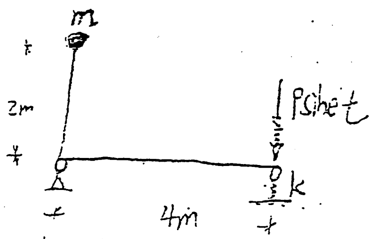
M图



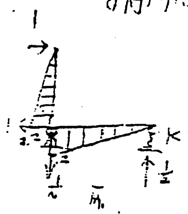


Q图

四题



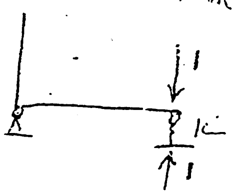
柔度法



$\delta_{11} = \frac{1}{EI}\left(\frac{1}{2}\times2\times2\times\frac{2}{3}\times2 + \frac{1}{2}\times4\times2\times\frac{2}{3}\times2\right) + \frac{1}{2}\times\frac{\frac{1}{2}}{k}$

$= \frac{8}{EI} + \frac{1}{4k} = \frac{\quad}{4EIk}$

$\omega = \sqrt{\frac{1}{m\delta_{11}}} = \sqrt{\frac{4EIk}{m(32k+EI)}}$　$\beta = \frac{1}{1-\frac{\theta^2}{\omega^2}}$



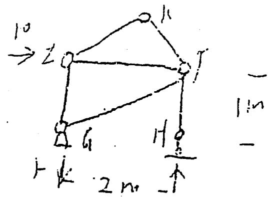
$\Delta_{1P} = 1\cdot\frac{\frac{1}{2}}{k} = \frac{1}{2k}$

$y_{max} = \beta\cdot y_{st} = \beta\cdot P\cdot\delta_{1P} = \frac{4EIk}{4EIk - m\theta^2(32k+EI)}\cdot P\cdot\frac{1}{2k}$

$= \frac{2EIP}{4EIk - m\theta^2(32k+EI)}$

(六) 有限刚性杆

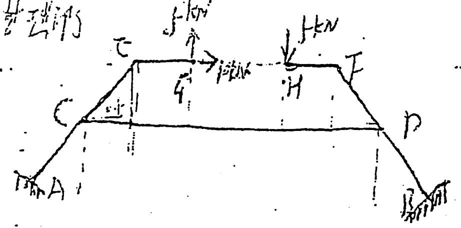
例: ① 先求支座反力



IK, KJ, ZG 为0杆

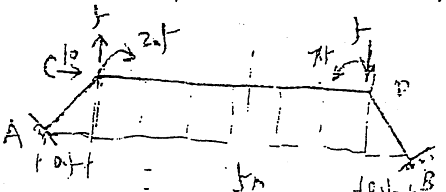
解 反力 $R_H = 5kN(\uparrow)$ $R_G = 5kN(\downarrow)$ $V_G = 10kN(\leftarrow)$

② 截开

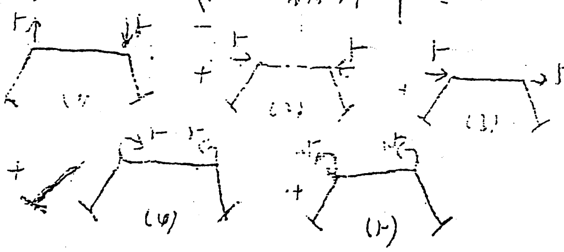


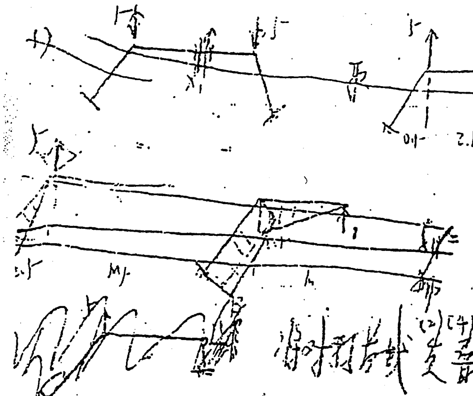
CG 与 DH 为悬臂结构, 内力易求

故只须分析 ACDB 超静定结构内力状况



取其反对称载荷作用

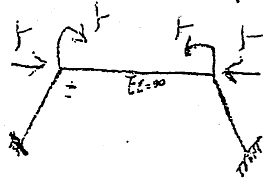




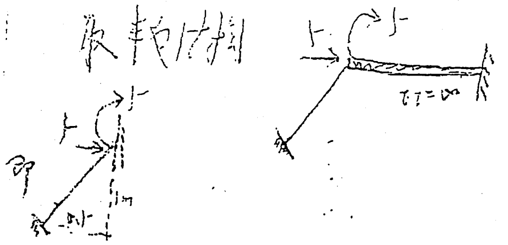
位移法

则对称载荷 只受轴力

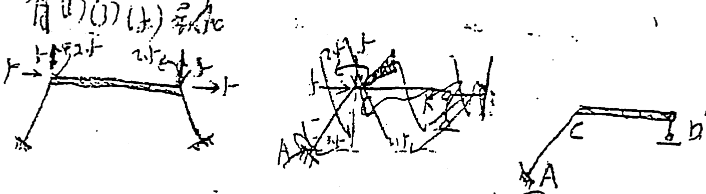
a.



取半结构

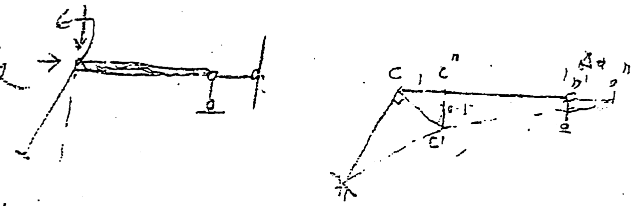


情(1)(2)(3)



另外注意 因无转动

本题C处只有



$D'D'' = CC'' = \Delta$, $C'C'' = \frac{\Delta}{2}$, $CC' = \frac{\sqrt{5}}{2}\Delta$

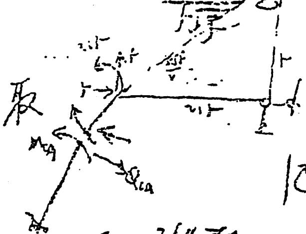
$$\varphi_c \approx tg\varphi_c = \frac{CC''}{C''D''} = \frac{CC''}{CD'} = \frac{\frac{\Delta}{2}}{2.5} = \frac{\Delta}{5}$$

$$\text{故 } M_{CA} = -4i\varphi_c - \frac{6i}{l}\Delta' = -\frac{4EI}{\frac{\sqrt{5}}{2}}\cdot\frac{\Delta}{5} - \frac{6EI}{\frac{5}{4}}\cdot\frac{\sqrt{5}}{2}\Delta$$

$$M_{AC} = -2i\varphi_A - \frac{6i}{l}\Delta' = -\frac{68EI}{5\sqrt{5}}\Delta$$

$$= -\frac{64EI}{5\sqrt{5}}\Delta$$

取



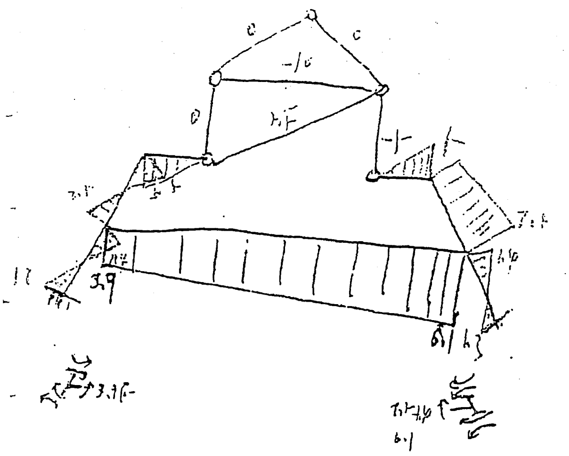
$\Sigma M_0 = 0$

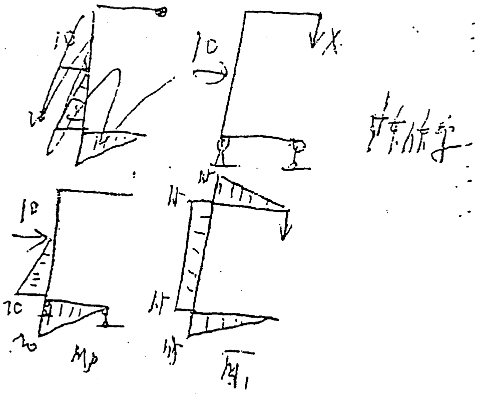
$$|Q_{CA}|\frac{\sqrt{5}}{2} + |M_{CA}| = 2.5 + 5\times 5$$

$$\text{即 } \frac{264EI}{25}\Delta\cdot\frac{\sqrt{5}}{2} + \frac{68EI}{5\sqrt{5}}\Delta = 25$$

$$\Rightarrow \Delta = \frac{75\sqrt{5}}{728EI} \Rightarrow M_{CA} = -1.9$$

$$M_{AC} = -1.3$$



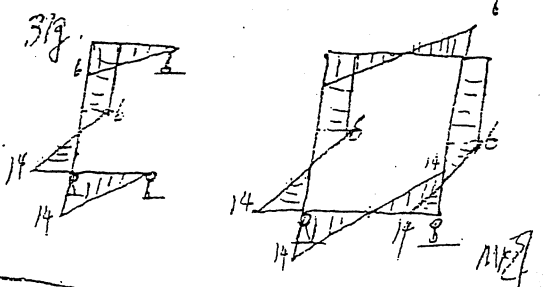


由 $\delta_{11}=\frac{1}{EI}\left(\frac{1}{2}\left(\frac{3}{2}\right)^2\times 2+\frac{3}{2}\times 4\times\frac{3}{2}\right)=\frac{45}{4EI}$

$\Delta_{1P}=\frac{1}{EI}\left(\frac{1}{2}\times 20\times 2\times\frac{3}{2}+\frac{1}{2}\times 20\times\frac{3}{2}\times\frac{2}{3}\times\frac{3}{2}\right)=\frac{45}{EI}$

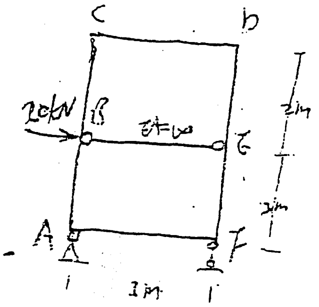
$\therefore X=-\frac{\Delta_{1P}}{\delta_{11}}=-4$

$\therefore M=\overline{M}_1X+M_P$

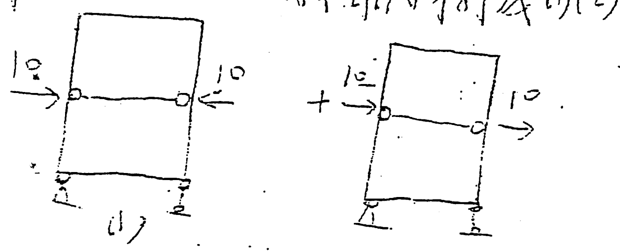


又

对称结构



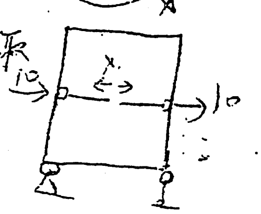
解：由题意 可将原图结构分解成(1)(2)



1) 因为二力杆BE, EA=∞, 故其它杆件内力为0, $N_{BE}=-10$ kN(压力)

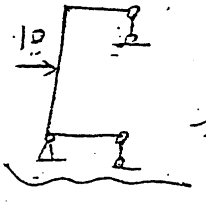
2) 对称结构在反对称荷载作用下

取半结构

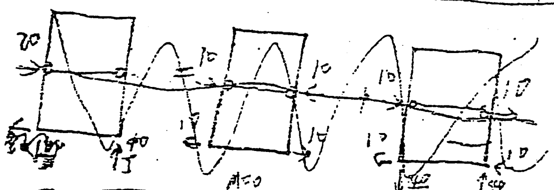


X1必为0

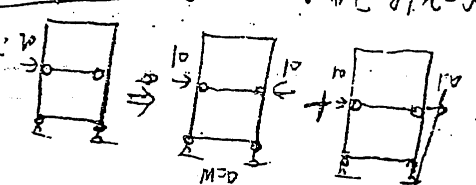
又再取半边结构

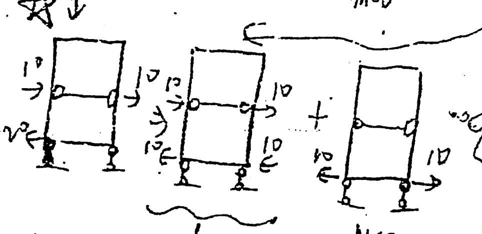


一次超静定结构



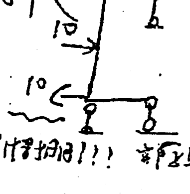
支座：





掌握此法

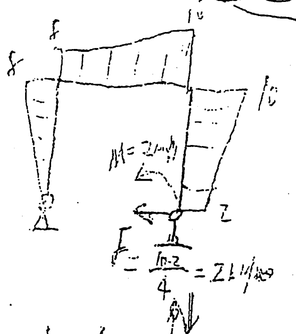
把对称结构→分解



结构!!! 学会了什么样的选择取!!反对称!!

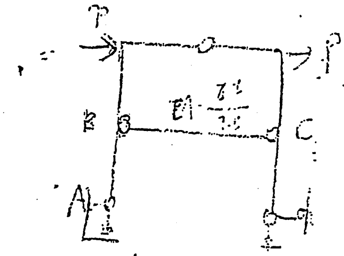
一九九八年

一、1.

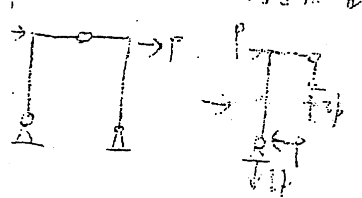


只在E处有顺时针方向的大小为2kN·m的力偶，E点水平向左的集中荷载2kN的作用.

2.



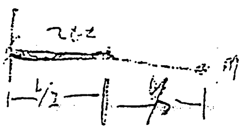
对称结构受反对称荷载 $\therefore N_{BC}=0$



$H_A=P(\leftarrow)$

$V_A=2P(\downarrow)$

3.



求M图

$$\delta_{11}=\frac{1}{EI}\cdot\frac{1}{3}\left(\frac{l}{2}\right)^3+\frac{1}{2EI}\left(\frac{1}{2}\times\frac{l}{2}\times\frac{l}{2}\times\frac{l}{2}+\frac{1}{2}\times\frac{l}{2}\times\frac{l}{2}\times\frac{\frac{l}{2}+l}{2}\right)$$

$$=\frac{3l^3}{16EI}$$

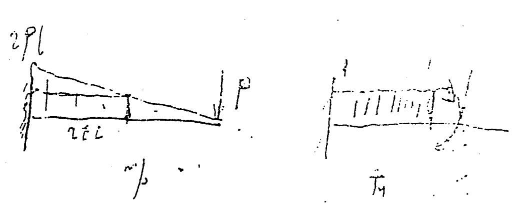
$$\omega=\sqrt{\frac{1}{m\delta_{11}}}=\sqrt{\frac{16EI}{3ml^3}}$$

当振幅 振幅 $A=\sqrt{y_0^2+\frac{v_0^2}{\omega^2}}$

即 $y_{max}=\sqrt{y_{st}^2+\frac{v_0^2}{\omega^2}}$ $\therefore v_0=\sqrt{\frac{2EI}{ml^3}}$

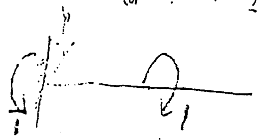
$=y_{max}-y_{st}$ (达到平衡位置为原点!!! 考虑是动位移!!)

二、1.



M_P $\bar{M}_1$

$$\Delta_1=\frac{1}{2EI}\left(1\times l\cdot\frac{Pl+2Pl}{2}\right)=\frac{3Pl^2}{4EI}(\downarrow)$$

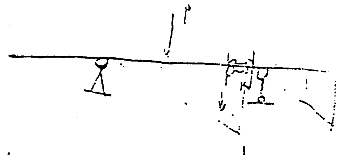


$$\Delta_2=-\sum \bar{R}\cdot c=-(-1)\cdot\varphi=\varphi(\downarrow)$$

$$\therefore \Delta=\Delta_1+\Delta_2=\frac{3Pl^2}{4EI}+\varphi \quad \therefore [B]$$

2. (?)

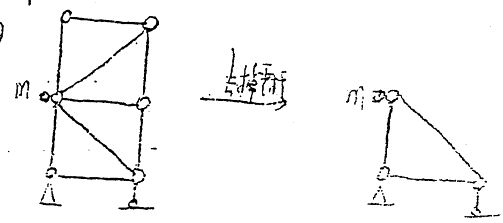
3.



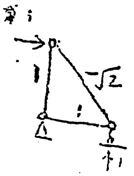
[B]

三、自振频率

(1)



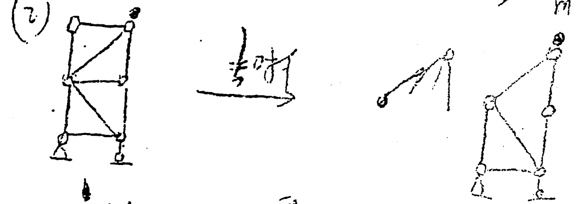
去掉零杆



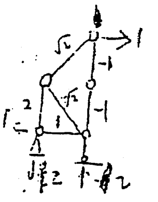
$$\delta_1=\frac{\sum\bar{F}^2l}{EA}=\frac{1}{EA}(1\cdot4+1\times4+2\times4\sqrt{2})=\frac{8+8\sqrt{2}}{EA}$$

$$\omega_1=\sqrt{\frac{1}{m\delta_1}}=\sqrt{\frac{EA}{m(8+8\sqrt{2})}}$$

(2)



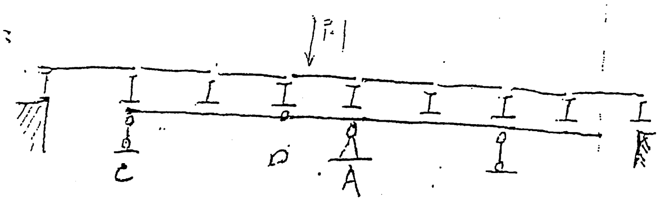
去零杆



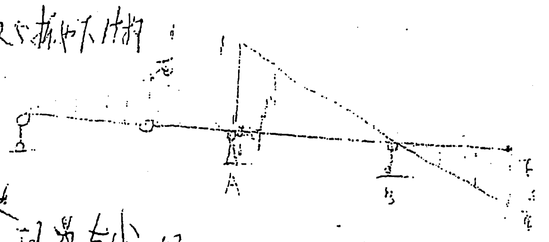
$$\delta_2=\frac{\sum\bar{F}^2l}{EA}=\frac{1}{EA}(1\times4\times3+2\times4\sqrt{2}\times2+4\times4)=\frac{28+16\sqrt{2}}{EA}$$

$$\omega_2=\sqrt{\frac{1}{m\delta_2}}=\sqrt{\frac{EA}{(28+16\sqrt{2})m}}$$

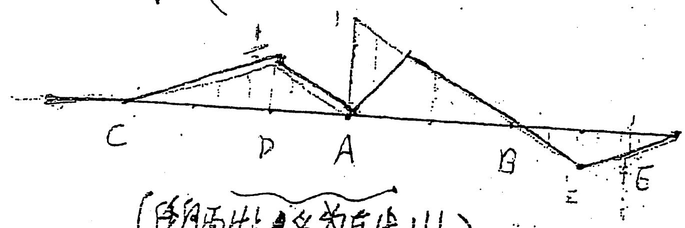
7)解：



先做C截面大作用

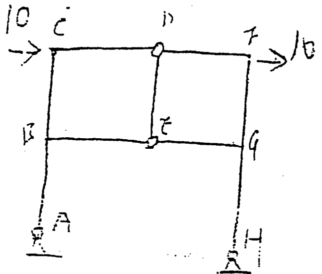


刚度
(结点)间为直线，故



(结点两侧影响线为直线!!!)

8)解



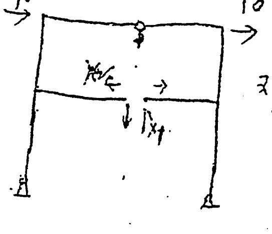
取 ⊢|⊣ $W=3m-2n-C_0-C$

$=3\cdot3-2\cdot4-0-4=-3$

对称结构在反对称荷载作用下

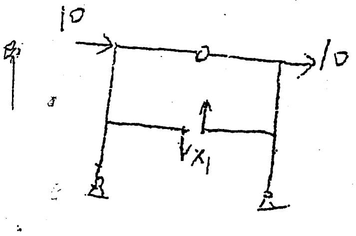
~~DE~~ $N_{DE}=0$

取

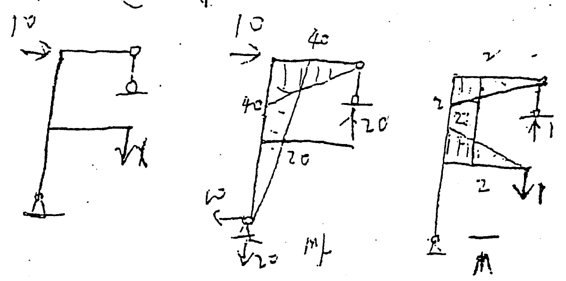


则 $X_2=0$

即



取半边结构



$$\delta_{11}=\frac{1}{EI}\left(\frac{1}{2}\times2\times2\times\frac{2}{3}\cdot2\times2+2\times2\times2\right)$$

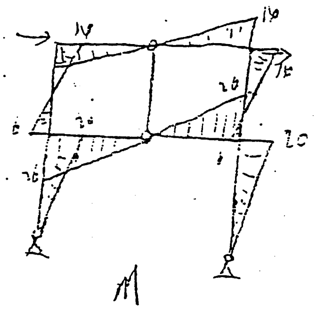
$$=\frac{40}{3EI}$$

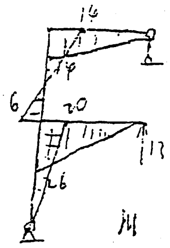
$$\Delta_{1P}=\frac{1}{EI}\left(\frac{1}{2}\times2\times2\times\frac{2}{3}\times40+2\times2\times\frac{20+40}{2}\right)$$

$$=\frac{520}{3EI}$$

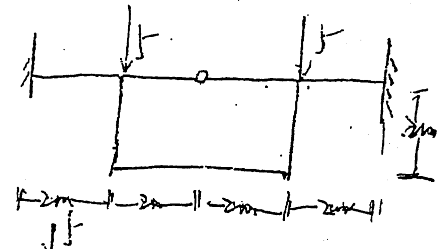
$$X_1=-\frac{\Delta_{1P}}{\delta_{11}}=-13$$

$M=\overline{M}_1\cdot X_1+M_P$

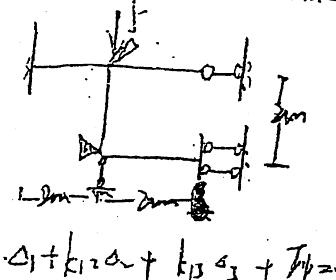




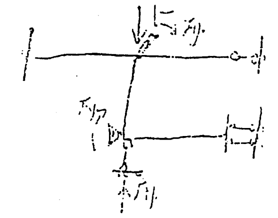
六解：



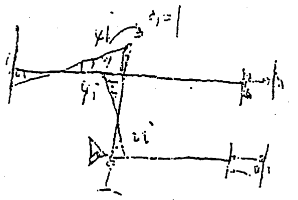
取



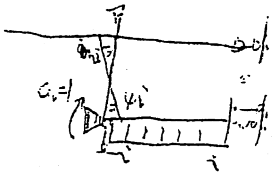
$$\begin{cases}k_{11}\Delta_1+k_{12}\Delta_2+k_{13}\Delta_3+F_{1P}=0\\k_{21}\Delta_1+k_{22}\Delta_2+k_{23}\Delta_3+F_{2P}=0\\k_{31}\Delta_1+k_{32}\Delta_2+k_{33}\Delta_3+F_{3P}=0\end{cases}$$



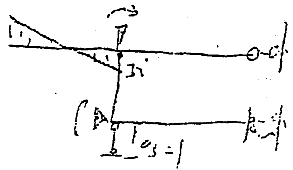
$F_{1P}=-15,\ F_{2P}=F_{3P}=0$



$k_{11}=8i$
$k_{21}=2i$
$k_{31}=-3i$



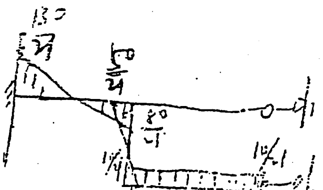
$k_{12}=2i$
$k_{22}=5i$
$k_{32}=0$

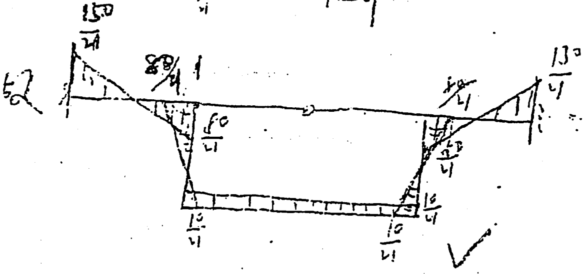


$k_{13}=-3i$
$k_{23}=0$
$k_{33}=3i$

$$\begin{cases}8i\Delta_1+2i\Delta_2-3i\Delta_3=0\\2i\Delta_1+5i\Delta_2=0\\-3i\Delta_1+3i\Delta_3-5=0\end{cases}\Rightarrow\begin{cases}\Delta_1=\frac{25}{21i}\\\Delta_2=-\frac{10}{21i}\\\Delta_3=\frac{20}{7i}\end{cases}$$

$M=\bar{M}_1\Delta_1+\bar{M}_2\Delta_2+\bar{M}_3\Delta_3$

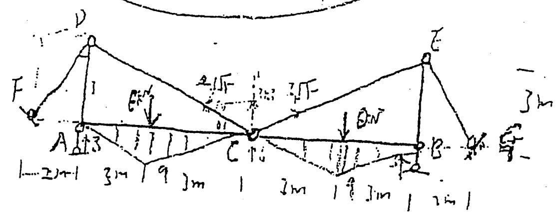




M图

一九九八年

一、解：



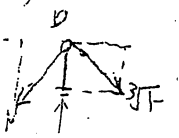
~~$V_A=3$~~ 结构对称，只有折杆中杆内力

$V_A=3$，取C点，$\Sigma F_y=0$

$N_{CD}\sin\alpha=N_{CD}\cdot\frac{1}{\sqrt5}=3$

$N_{CD}=N_{CE}=3\sqrt5$ (拉)

取D点平衡 $\Sigma F_x=0$



$N_{DF}\cdot\frac{2}{\sqrt{13}}=N_{CD}\cdot\frac{2}{\sqrt5}=6$

$N_{DF}=3\sqrt{13}$ (拉)　$\Sigma F_y=0$，$N_{AD}=-(N_{CD}\frac{1}{\sqrt5}+N_{DF}\frac{3}{\sqrt{13}})$

$=-(3+9)=-12\ kN$ (压)

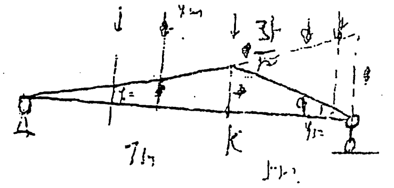
二、内力各数据，三力杆：

$N_{CD}=N_{CE}=3\sqrt5\ kN$ (拉力)　$N_{AD}=N_{BE}=-12\ kN$ (压力)

$N_{DF}=N_{EG}=3\sqrt{13}\ kN$ (拉力)

零杆在图中画出。✓

二、解：



M_K影响线

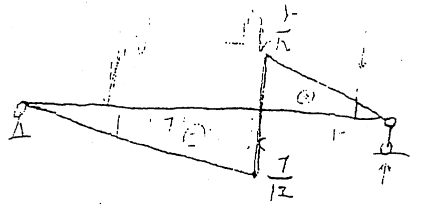
将移动荷载作用于K点上，是M_K最不利荷载位置。

$y_1=\frac{5}{4}$，$y_3=\frac{7}{12}$

$M_{max}=P_1y_1+P_2y_2+P_3y_3=1\times\frac{5}{4}+\ldots\times\frac{7}{12}$

$=\frac{91}{4}\ kN\cdot m$

(只限定汽车荷载向左向右移动时的 P_2-P_3)



Q_K 影响线

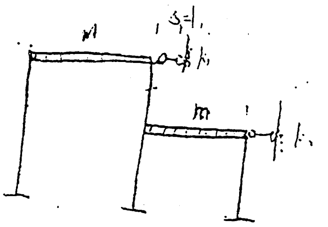
P_1 放在K处

$V_{max} = 5\times\frac{5}{12} + 5\times\frac{5}{12}\times\frac{1}{5} = \frac{5}{2}$

K处

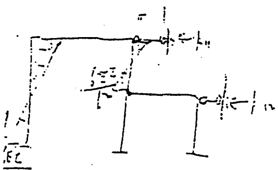
$V_{min} = -\left(5\times\frac{7}{12} + 5\times\frac{7}{12}\times\frac{3}{7}\right) = -\frac{25}{6}$ ✓

六、解：



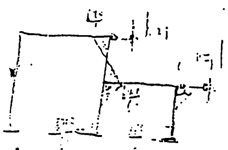
由这种结构的受力变形特点

$\Delta_1 = 1$ 时



$k_{11} = \frac{3EI}{2l^3} + \frac{12EI}{l^3} = \frac{27EI}{2l^3}$

$k_{12} = -\frac{12EI}{l^2} = k_{21}$



$k_{22} = \frac{12EI}{l^3}\times 3 = \frac{36EI}{l^3}$

由 $\begin{vmatrix} k_{11} - m_1\omega^2 & k_{12} \\ k_{21} & k_{22} - m\omega^2 \end{vmatrix} = 0$

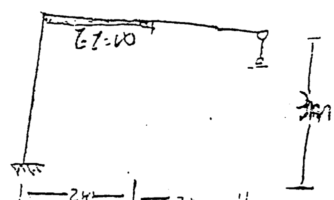
$\Rightarrow \omega^2_{1,2} = \frac{1}{2}\left(\frac{k_{11}}{m_1} + \frac{k_{22}}{m_2}\right) \pm \sqrt{\left[\frac{1}{2}\left(\frac{k_{11}}{m_1} + \frac{k_{22}}{m_2}\right)\right]^2 - \frac{k_{11}k_{22} - k_{12}k_{21}}{m_1 m_2}}$

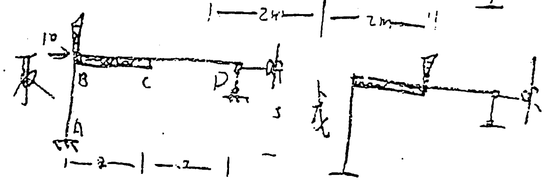
$\Rightarrow \omega_1 = 2.8817\sqrt{\frac{EI}{ml^3}}$

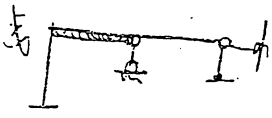
$\omega_2 = 6.4186\sqrt{\frac{EI}{ml^3}}$ ✓

四）刚架〔无限刚性杆题〕

五 解：



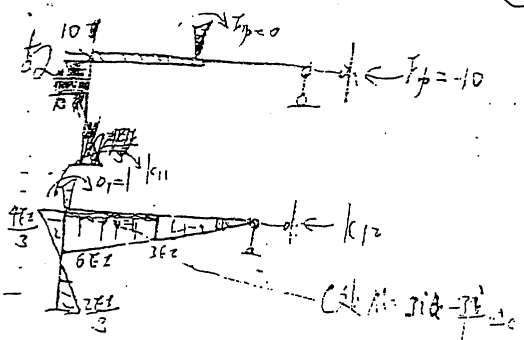


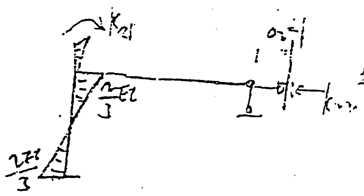


Δ 0.8.0.0

三种情况作用相同，因为三者所受约束 [illegible]

故 $\theta_B = \theta_C$ 故 $\Delta_C = \theta_C \cdot l_{BC}$





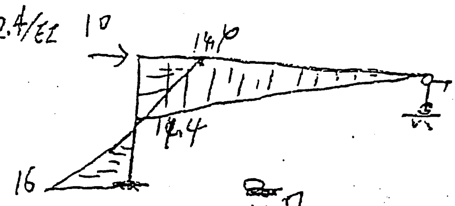
$\frac{3EI}{2} - \frac{3EI}{2\times 2}(2) = 3EI$

$k_{11} = \left(6+\frac{4}{3}\right)EI$，$k_{12} = -\frac{2}{3}EI = k_{21}$

$k_{22} = \frac{4}{9}EI$

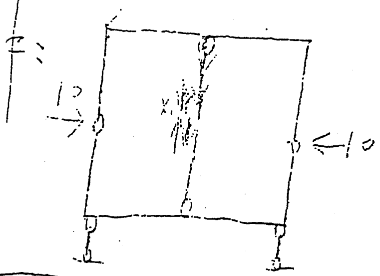
$\begin{cases} k_{11}\Delta_1 + k_{12}\Delta_2 + F_{1P} = 0 \\ k_{21}\Delta_1 + k_{22}\Delta_2 + F_{2P} = 0 \end{cases} \Rightarrow \begin{cases} \frac{22}{3}\Delta_1 - \frac{2}{3}\Delta_2 = 0 \\ -\frac{2}{3}\Delta_1 + \frac{4}{9}\Delta_2 = \frac{10}{EI} \end{cases}$

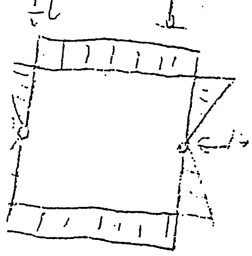
$\begin{cases} \Delta_1 = \frac{45}{19EI} = 2.4/EI \\ \Delta_2 = 26.4/EI \end{cases}$

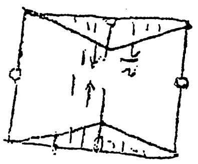


M图

1)







★ $+\frac{l}{EA}$ 轴力杆截断作拉力时 也有位移影响 千万别忽略!

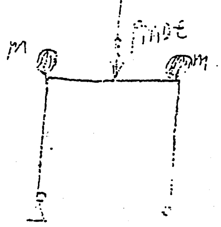
$$\delta_{11}=\frac{1}{EI}\cdot\left(\frac{1}{2}\times 2l\times\frac{l}{2}\times\frac{2}{3}\times\frac{l}{2}\times 2\right)=\frac{l^3}{3EI}+\frac{l}{EA}$$

$$\Delta_{1P}=-\frac{1}{EI}\cdot\frac{1}{2}\times 2l\times\frac{l}{2}\times\frac{1}{2}l\times 2=-\frac{Fl^3}{2EI}$$

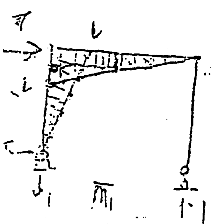
$$\cancel{X_1=\frac{-\Delta_{1P}}{\delta_{11}}=\frac{11}{2}}$$

$$X_1=-\frac{\Delta_{1P}}{\delta_{11}}=\frac{11FA\cdot l^2}{2Al^2+6I}$$

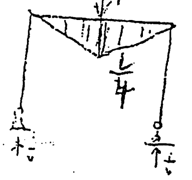
2)



$$y(t)=-2m\ddot{y}(t)\cdot\delta_{11}+P(t)\,\delta_{12}$$



$$\delta_{11}=\frac{2l^3}{3EI},$$



$$\delta_{12}=\frac{l^3}{16EI}$$

2) $\omega=\sqrt{\frac{1}{2m\delta_{11}}}=\sqrt{\frac{3EI}{4ml^3}}$

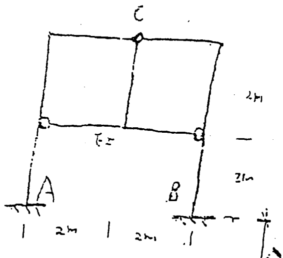
3) $y_{max}=\beta\cdot y_{st}=\beta P\delta_{12}$

$$\beta=\frac{1}{1-\frac{\theta^2}{\omega^2}}=\frac{3EI}{3EI-4m\theta^2l^3}$$

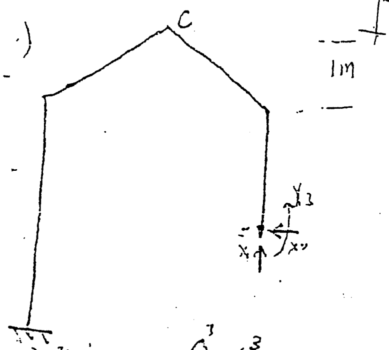
$$y_{max}=\frac{3Pl^3}{48EI-64m\theta^2l^3}$$

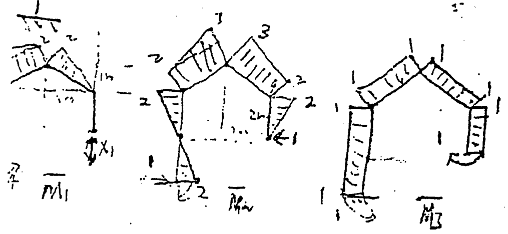
例：

1)



2)





$t_0=\frac{10-10}{2}=0 \qquad \Delta t=10-(-10)=20$

$\Delta_t=\alpha t_0\int\bar{F}_N ds+\frac{\alpha\Delta t}{h}\int\bar{M}ds$

$\Delta_{1t}=-\frac{\alpha\cdot 20}{h}\cdot\left(4\times4+\frac{4+2}{2}\times\sqrt{5}+\frac{1}{2}\times2\times\sqrt{5}\right)=$

$\Delta_{2t}=-\frac{\alpha\cdot 20}{h}\left(\frac{1}{2}\times2\times2+\frac{3+2}{2}\times\sqrt{5}\times2+(1\times1)\right)=$

$\Delta_{3t}=-\frac{\alpha\cdot 20}{h}\left[1\times(2+\sqrt{5}+\sqrt{5}+4)\right]=$

$\delta_{11}=\frac{1}{EI}\left[\frac{1}{2}\times2\sqrt{5}\times\frac{2}{3}\times2+2\times\sqrt{5}\times3+\frac{1}{2}\times2\sqrt{5}\left(\frac{2}{3}\times2+4\right)+4\times4\times4\right]$

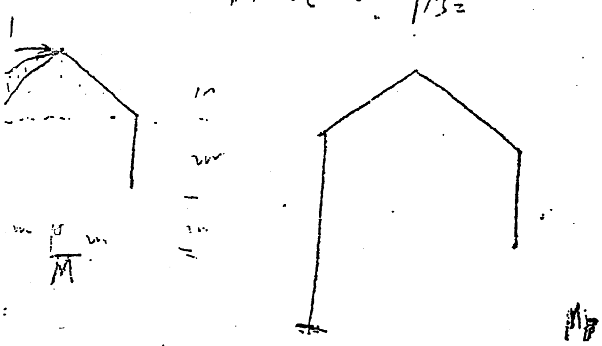
$=\delta_{21}=\frac{1}{EI}\left[\frac{1}{2}\times\sqrt{5}\times2\times\left(\frac{2}{3}\times1+3\right)+2\times\sqrt{5}\times3+\frac{1}{2}\times\sqrt{5}\times1\times\left(2+\frac{1}{3}\times2\right)\right]=$

$=\frac{1}{EI}\left[\frac{1}{2}\times2\times2\times\frac{2}{3}\times2+2\sqrt{5}\times\frac{5}{2}+\frac{1}{2}\times1\times\sqrt{5}\times\left(\frac{2}{3}\times1+2\right)\right]=$

$=\delta_{32}=\frac{1}{EI}\left[\frac{1}{2}\times2\times2\times1+\frac{3+2}{2}\times\sqrt{5}\times1\times2+0\right]=$

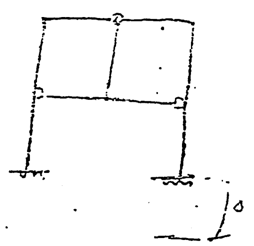
$=\frac{1}{EI}\left[1\times2\times1+1\times\sqrt{5}\times1\times2+1\times4\times1\right]=$

$$\left.\begin{array}{l}\delta_{11}X_1+\delta_{12}X_2+\delta_{13}X_3+\Delta_{1t}=0\\ \delta_{21}X_1+\delta_{22}X_2+\delta_{23}X_3+\Delta_{2t}=0\\ \delta_{31}X_1+\delta_{32}X_2+\delta_{33}X_3+\Delta_{3t}=0\end{array}\right\}\Rightarrow\begin{cases}X_1=\\X_2=\\X_3=\end{cases}$$



$\bar{M}$ M_P

超静定结构支座移动 引起位移不能 (?)

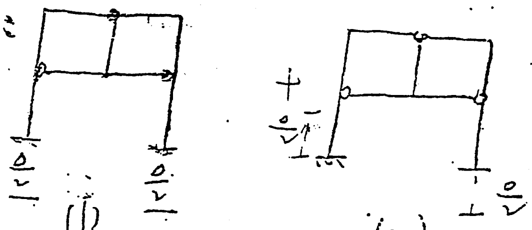


△ 支座移动下的位移计算

因结构对称

利用 $\delta_i X_i+\Delta_i=0$ 计算

因结构对称，支座移动按荷载一样分解为：



(1)中 $\Delta_{C1}=\frac{\Delta}{2}(\downarrow)$ (2)中 $\Delta_C=0$

$\therefore \Delta_C=\frac{\Delta}{2}(\downarrow)$

000年）答案

解：由题可知 a、e两杆为零杆

有 $N_a=N_e=0$

且有 $N_b=30kN$（压力）

取C点：$\Sigma F_y=0$　$N_b=N_d\cdot\frac{4}{5}$

$N_d=\frac{5}{4}N_b=37.5kN$（压力）

以整体为对象

$R_B=10kN$　$R_A=40kN$

截面取截面I-I，取左面分析

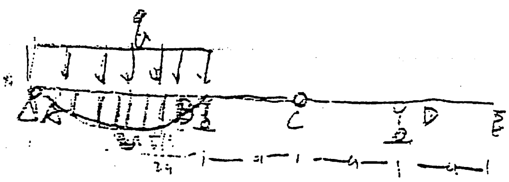
由 $\Sigma F_y=0$．$N_{cy}=40-30=10kN$（↓）

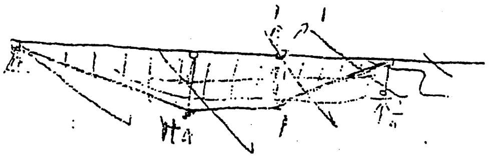
故 $N_c=N_{cy}\cdot\frac{2\sqrt{13}}{4}=10\times\frac{\sqrt{13}}{2}=5\sqrt{13}$（拉力）

归纳：$N_a=0$，$N_e=0$，$N_b=50kN$（压力）

$N_c=5\sqrt{13}kN$（拉）　$N_d=37.5kN$（压力）

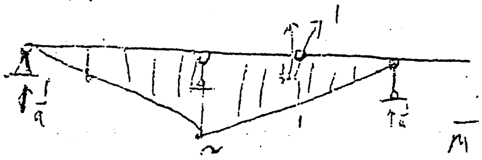
解：1)





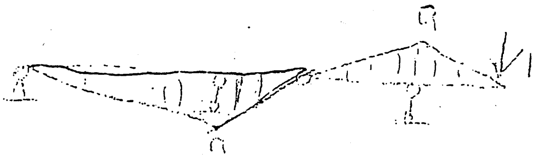
~~得 $\Delta_C=\frac{1}{EI}(\frac{2}{3}\cdot 2a\cdot\frac{1}{2}qa^2\times\frac{1+a}{2})=\frac{qa^3(1+a)}{3EI}$（↓）~~

2)



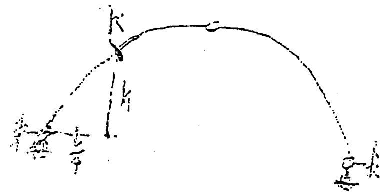
$\Delta_C=\frac{1}{EI}\cdot(\frac{2}{3}\cdot 2a\times\frac{1}{2}qa^2\times 1)=\frac{2qa^3}{3EI}$（↓）

2)

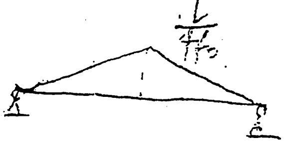


$\Delta_C=\frac{1}{EI}(\frac{2}{3}qa\cdot\frac{1}{2}qa^2\cdot\frac{a}{2})=\frac{qa^4}{3EI}$（↓）

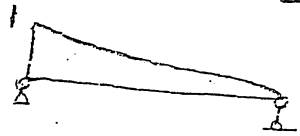
三、解：



$H_A=\frac{M_C^0}{f_0}$，



R_A

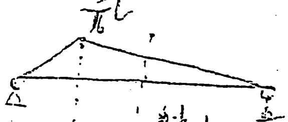


~~由 $M_K=R_A\cdot\frac{l}{4}-H_Ah$~~

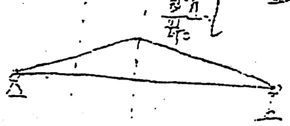
~~$M_K=R_A\cdot\frac{l}{4}-H_Ah$~~

$M_K=M_0-H_Ah$

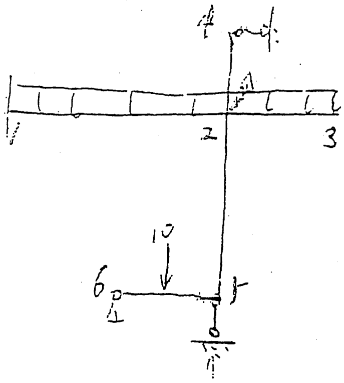
$\bar{M}_0$

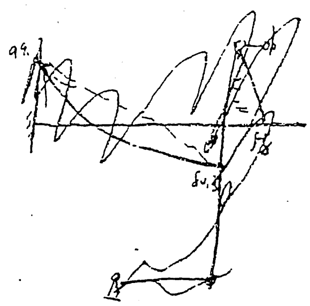


H_Ah

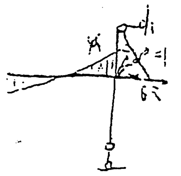


回转？



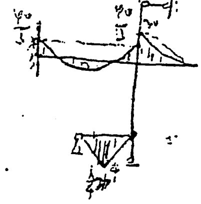


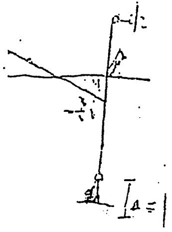
$F_{1P}=-\frac{20}{3}$, $F_{2P}=-(1+20+\frac{21}{2})=-41$



$k_{11}=10i$

$k_{12}=-\frac{3}{2}i$





$k_{21}=-\frac{3}{2}i$

$k_{22}=\frac{3}{8}i$

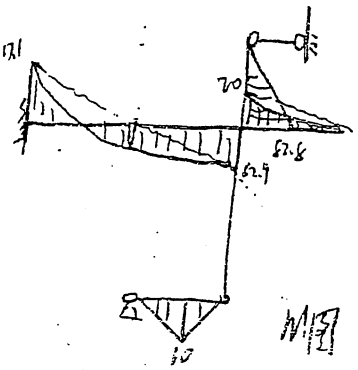
$\theta_2=1$

$10i\theta_1-\frac{3}{2}i\theta_2-\frac{20}{3}=0$

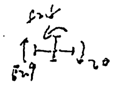
$-\frac{3}{2}i\theta_1+\frac{3}{8}i\theta_2-41=0$

$\Rightarrow \begin{cases}\theta_1=\frac{13.8}{i}\\ \theta_2=87.6/i\end{cases}$

这是另一种结构这么用时 这样用了时，只要之加 $\bar{M}_2$ 上 ∵ $M=\bar{M}_1\theta_1+\bar{M}_2\theta_2+M_P$



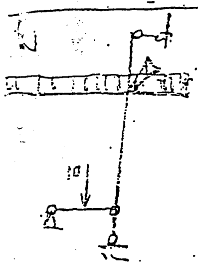
M图



$\Sigma M_1=0$.

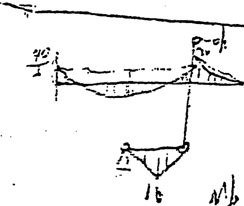
？ X

那里出错了？因为定向计算的话支座处变形不平衡！

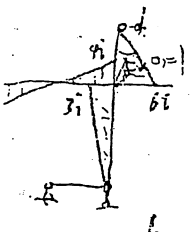


$F_{1P}=-\frac{20}{3}$

$F_{2P}=-41$

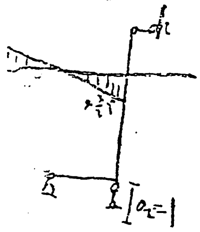


M_P



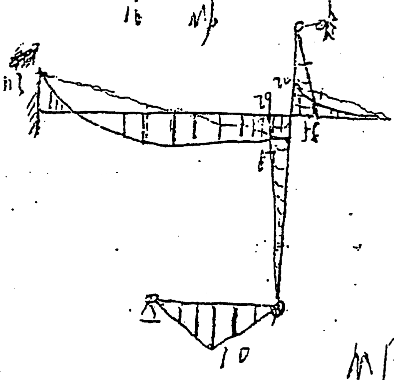
$k_{11}=13i$

$k_{21}=-\frac{3}{2}i$

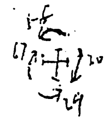


$k_{12}=-\frac{3}{2}i$

$k_{22}=\frac{3}{8}i$



M图



$\Sigma M_2=0$.

✓

$13i\theta_1-\frac{3}{2}i\theta_2-\frac{20}{3}=0$

$-\frac{3}{2}i\theta_1+\frac{3}{8}i\theta_2-41=0$

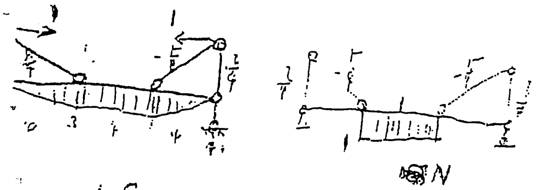
$\Rightarrow \begin{cases}\theta_1=\frac{29}{3i}\\ \theta_2=\frac{238}{3i}\end{cases}$

$M=\bar{M}_1\theta_1+\bar{M}_2\theta_2+M_P$

2001

X 2.X

$t_0 = \frac{t_1+t_2}{2} = \frac{10+0}{2} = 5$. $\Delta t = 0-10 = -10$ ℃



$1 = \alpha t_0 \int \bar N ds + \frac{\alpha \Delta t}{h}\int \bar M ds$

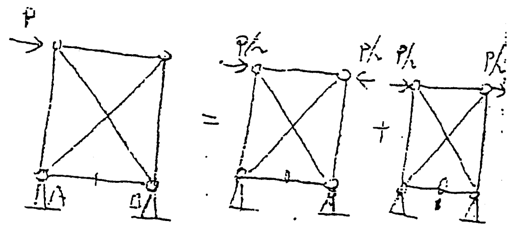
$= \alpha \cdot 5 \cdot (-\frac{5}{4}\times 5\times 2 + \frac{3}{4}\times 3\times 2 \mp 1\times 4)$

$-\frac{\alpha\cdot 10}{h}\cdot(\frac{1}{2}\times 3\times 4\times 2 + 3\times 4)$

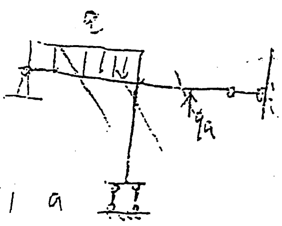
$= -60\alpha - 400\alpha = -460\alpha$ (2xx)

3.(A)

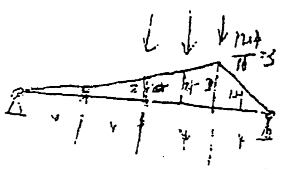
(X)



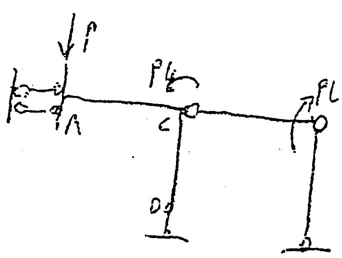
AB = 0



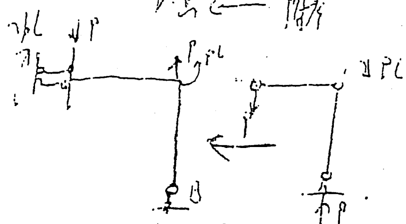
[C]



$= 20(2+3+2.5) = 150$ kN·m [D]



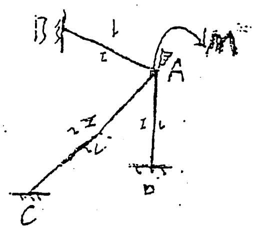
节点一断



[D]

$\Sigma M_B = 0 \Rightarrow M_A = 2Pl$

4.



$i = \frac{EI}{l}$ $i_{AB} = i_{AC} = i_{AD}$

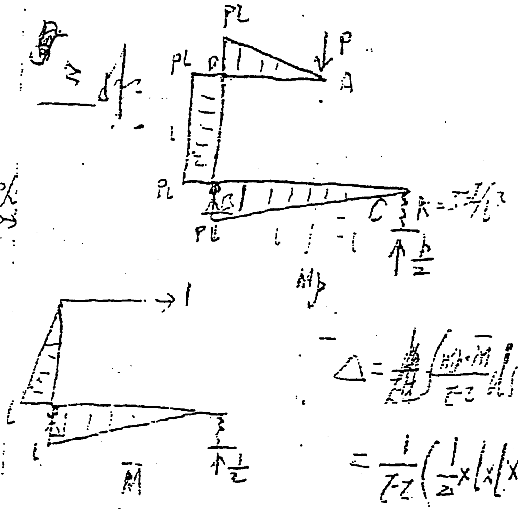
$M_{BA} = 2i\theta_A$ $M_{CA} = 2i\theta_A$ $M_{DA} = 2i\theta_A$

$4i\theta_A \times 3 = M \Rightarrow 2i\theta_A = M/6$?

[B]

[用转角位移法]!! $i_{AB} = i_{AC} = i_{AD}$?

5. 解:



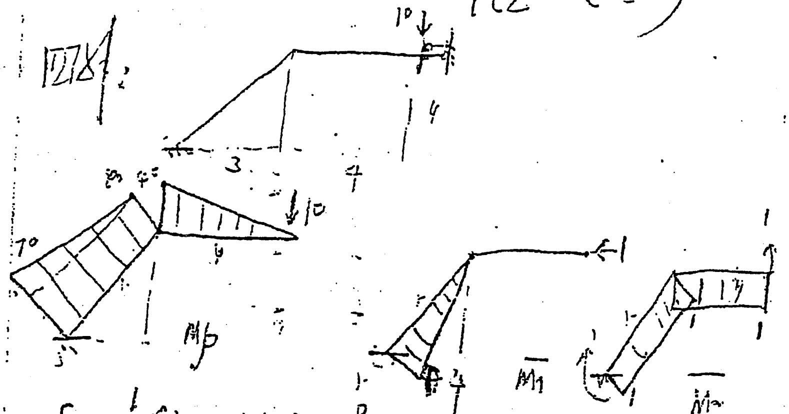
$\Delta = \sum\int \frac{M_P \bar M}{EI} ds - \sum \bar R c$

$= \frac{1}{EI}(\frac{1}{2}\times l\times l\times Pl + \frac{1}{2}\times l\times 2l\times\frac{2}{3}\times Pl) + \frac{P}{2}\cdot\frac{\frac{1}{2}}{k}$

$= \frac{7Pl^3}{6EI} + \frac{Pl^3}{12EI}$

$= \frac{5Pl^3}{4EI}$ (→)

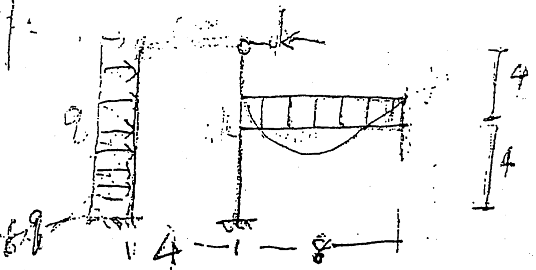
6. 解:



$\delta_{22} = \frac{1}{EI}(1\times 4\times 1 + 1\times 1\times 3) = \frac{9}{EI}$

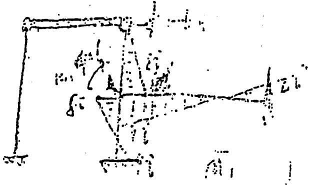
$\Delta_{2P} = -\frac{1}{EI}(\frac{1}{2}\times 4\times 40\times 1 + 1\times 1\times\frac{70+80}{2}) = -\frac{315}{EI}$

解：



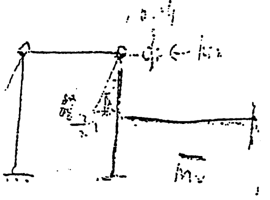
$F_{1P} = -\frac{1}{12} q l^2 = -\frac{16}{3} q$

$F_{2P} = -\frac{3}{8} q l = -3q$



$k_{11} = 8i + 4i + 6i = 18i$

$k_{21} = -\frac{3}{2} i$

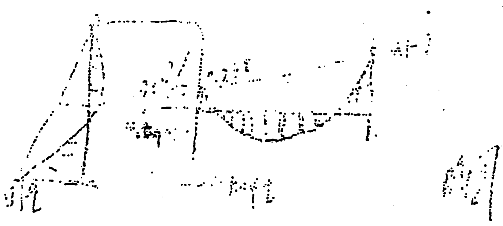


$k_{12} = -\frac{3}{2} i$

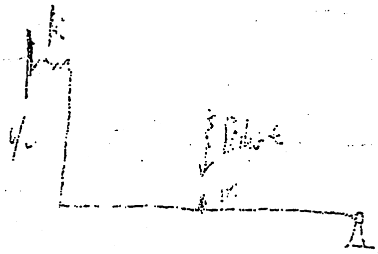
$k_{22} = \frac{15}{16} i$

$$\begin{cases} 18i\,\theta_1 - \frac{3}{2} i\,\theta_2 - \frac{16}{3} q = 0 \\ -\frac{3}{2} i\,\theta_2 + \frac{15}{16} i\,\theta_2 - 3q = 0 \end{cases} \Rightarrow \begin{cases} \theta_1 = \\ \theta_2 = \end{cases}$$

$M = \bar{M}_1 \theta_1 + \bar{M}_2 \theta_2 + M_P$



M图



$\bar{M}_1$

$\omega = \sqrt{\frac{1}{m\delta_{11}}}$

$\delta_{11} = \frac{1}{EI}\left[\ldots\right] = \frac{5l^3}{24EI}$

$\omega = \sqrt{\frac{1}{m\delta_{11}}} = \sqrt{\frac{24EI}{5ml^3}}$

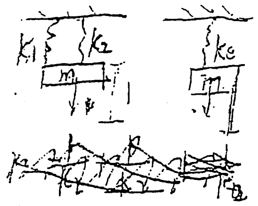
$\theta = \frac{2\pi n}{60} = \frac{2\pi \times 300}{60} = 31.4$

$\beta = \frac{1}{1-\frac{\theta^2}{\omega^2}}$

$y_{max} = (\beta P + mg)\,\delta_{11} = \checkmark$

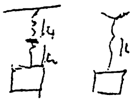
注意别把EI的E写错了！这种都是白扣的!!!

2002

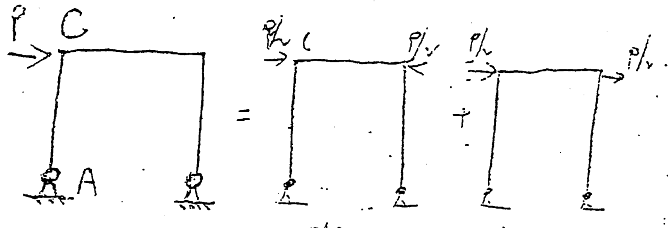


并联：$k_e=k_1+k_2$ [A]

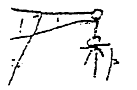
串联：



$\frac{1}{k_e}=\frac{1}{k_1}+\frac{1}{k_2}$ 即 $k_e=\frac{k_1k_2}{k_1+k_2}$

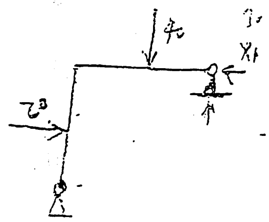


$M_{C_1}=0$ $M_{C_2}=?$

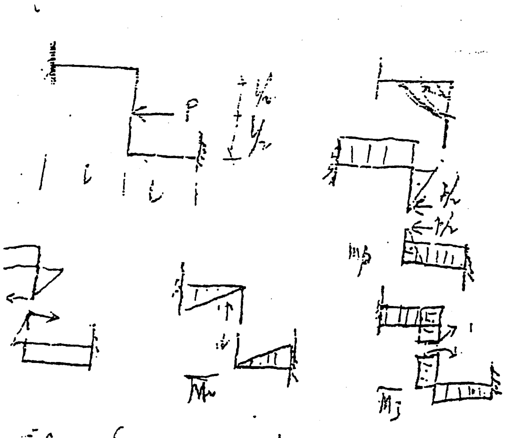


[C]

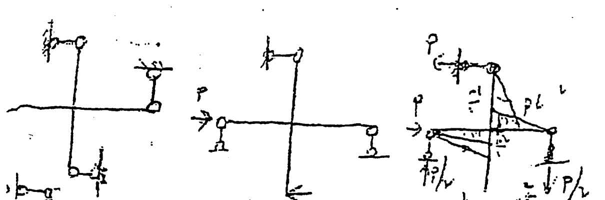
$P/2$ l $M_A=Pl/2$ (↑)

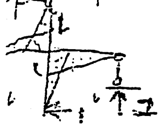


$X=-\frac{\Delta_{1P}}{\delta_{11}}$ 不等于 [A]



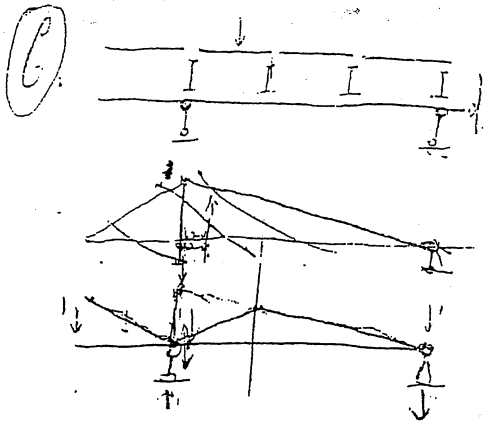
$=0$, $\delta_{31}=0$, $\Delta_{1P}=0$, $\delta_{12}<0$ [D]





$\delta_{11}=\frac{1}{EI}\frac{l^3}{3}\times 4$, $\Delta_{1P}=\frac{-1}{EI}\left(\frac{1}{2}\times l\times l\times\frac{2}{3}\times\frac{Pl}{2}\times 2+\frac{1}{2}\times l\times l\times\frac{2}{3}\times Pl\right)$

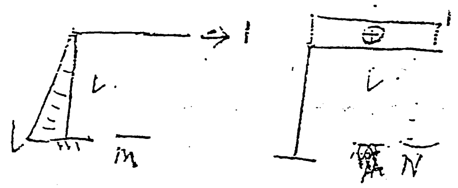
$X_1=\frac{1}{2}P$. $=\frac{-1}{EI}\frac{2Pl^3}{3}$ [D]



[B]

二

1. (X)
2. (X)
3. (X) N不为零 $\neq 0$
4. 解 $t_0=\frac{3}{2}t$ $\Delta t=t-2t=-t$



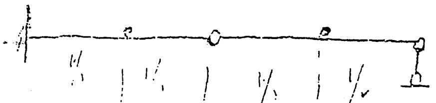
$\Delta_{CH}=\alpha\cdot t_0\int\bar N\,dx+\frac{\alpha\Delta t}{h}\int\bar M\,ds$

$=\alpha\frac{3}{2}t\times1\times l+\alpha\cdot t\frac{\times0}{l}\times\frac{1}{2}\times l^2$

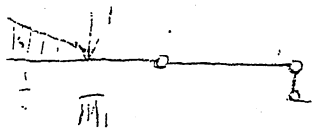
$=\frac{3}{2}\alpha\cdot tl+\cancel{\ \ }+\alpha t$.

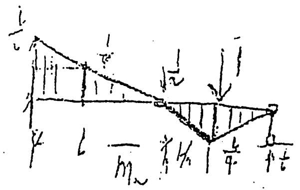
∴ (X)

解：



两个自由度





$$\delta_{11}=\frac{1}{EI}\left(\frac{1}{3}\times\left(\frac{l}{2}\right)^3\right)=\frac{l^3}{24EI}$$

$$\delta_{12}=\delta_{21}=\frac{1}{EI}\left[\frac{1}{2}\times\frac{l}{2}\times\frac{l}{2}\times\left(\frac{l}{4}+\frac{2}{3}\times\frac{l}{4}\right)\right]=\frac{5l^3}{96EI}$$

$$\delta_{22}=\frac{1}{EI}\left(\frac{1}{2}\times l\times\frac{l}{2}\times\frac{2}{3}\times\frac{l}{2}+\frac{1}{2}\times\frac{l}{2}\times\frac{l}{4}\times\frac{2}{3}\times\frac{l}{4}\times 2\right)=\frac{5l^3}{48EI}$$

$$\text{由}\begin{vmatrix} m_1\delta_{11}-\frac{1}{\omega^2} & m_2\delta_{12} \\ m_1\delta_{21} & m_2\delta_{22}-\frac{1}{\omega^2}\end{vmatrix}=0$$

$\Rightarrow$ 令 $\lambda=\dfrac{1}{\omega^2}$

$$\lambda_i=\frac{(m_1\delta_{11}+m_2\delta_{22})\pm\sqrt{(m_1\delta_{11}+m_2\delta_{22})^2-4m_1m_2(\delta_{11}\delta_{22}-\delta_{12}\delta_{21})}}{2}$$

$$=\frac{ml^3}{96EI}(7\pm\sqrt{29})=\begin{matrix}0.1317\frac{ml^3}{EI}\\ 0.01975\frac{ml^3}{EI}\end{matrix}$$

$\omega_1^2=7.4794\frac{EI}{ml^3}$ $\qquad \omega_1=2.735\sqrt{\frac{EI}{ml^3}}$

$\omega_2^2=\frac{82.118EI}{ml^3}$ $\qquad \omega_2=9.062\sqrt{\frac{EI}{ml^3}}$

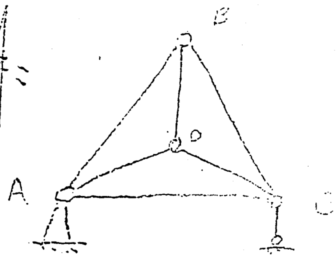
$$\text{由}\left(m_1\delta_{11}-\frac{1}{\omega^2}\right)Y_1+m_2\delta_{12}Y_2=0$$

$$m_2\delta_{21}Y_1+\left(m_2\delta_{22}-\frac{1}{\omega^2}\right)Y_2=0$$

$$\Rightarrow\frac{Y_{11}}{Y_{21}}=\frac{-m_2\delta_{12}}{m_1\delta_{11}-\frac{1}{\omega_1^2}}=\frac{0.5659}{1}$$

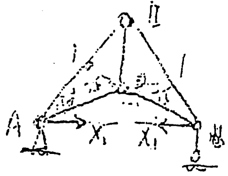
$$\frac{Y_{12}}{Y_{22}}=\frac{-m_2\delta_{12}}{m_1\delta_{11}-\frac{1}{\omega_2^2}}=\frac{-1.7662}{1}$$

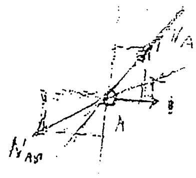
四、解：

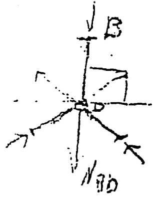


$m=6$ $\quad h=2+2+2+2=8$ $\quad C_0=3$

$W=18-16-3=-1$ 一次超静定结构







$$\overline{N}_{AD}\cos 30^\circ=1+\overline{N}_{AB}\sin 30^\circ$$

$$\overline{N}_{AD}\cdot\sin 30^\circ=\overline{N}_{AB}\cdot\cos 30^\circ$$

$$\Rightarrow\begin{cases}\overline{N}_{AD}=-\sqrt{3}\ (\text{压})\\ \overline{N}_{AB}=1\ (\text{拉})\end{cases}\qquad \text{同理}\ \begin{matrix}\overline{N}_{CD}=-\sqrt{3}\ (\text{压})\\ \overline{N}_{BC}=1\ (\text{拉})\end{matrix}$$

$$\overline{N}_{BD}=-\sqrt{3}\ (\text{压})$$

$$\delta_{11}=\frac{\sum\overline{N}^2l}{EA}=\frac{1}{EA}\left(1\times l\times 3+3\times\frac{\sqrt{3}}{3}l\times 3\right)=\frac{(3+3\sqrt{3})l}{EA}$$

$$\delta_{11}X_1+\Delta_{1t}=0$$

$$\Delta_{1t}=\alpha t\int\overline{N}ds=\alpha t\left[(-\sqrt{3})\times\frac{\sqrt{3}}{3}l\times 3\right]$$

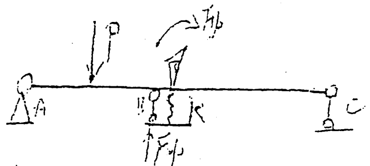
$$=-3\alpha tl$$

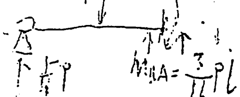
$$X_1=\frac{3\alpha tl}{\frac{3+3\sqrt{3}}{EA}l}=\frac{\alpha tEA}{1+\sqrt{3}}$$

$$N_{AB}=1\cdot X_1=\frac{\alpha tEA}{1+\sqrt{3}}=N_{BC}=N_{AC}$$

$$N_{AD}=N_{BD}=N_{CD}=-\frac{\sqrt{3}}{1+\sqrt{3}}\alpha tEA$$

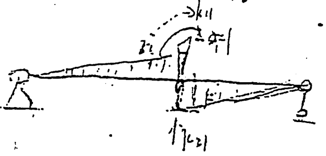
8





$\Rightarrow F_{1p}=\frac{3}{16}Pl$

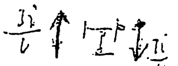
$F_{2p}=-\frac{11}{16}P$

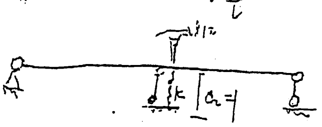


$\frac{EI}{l}=i$

$k_{11}=6i$

$k_{21}=\cancel{\ }0$



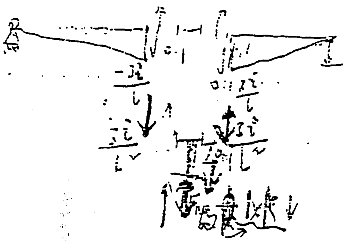


$k_{12}=0.$

$\cancel{k_{22}=}$

$k_{22}=\frac{6i}{l^2}+\cancel{\ }k=\frac{11EI}{l^3}$

$=\frac{11i}{l^2}$

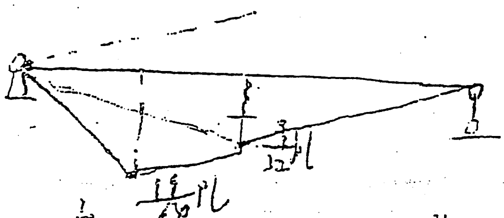


$k_{11}\Delta_1+k_{12}\Delta_2+F_{1p}=0$

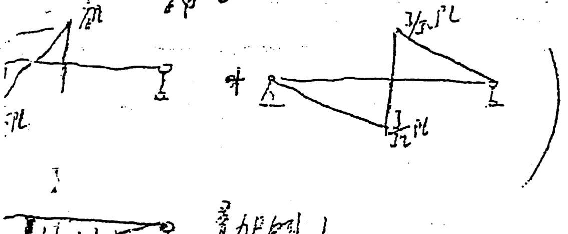
$k_{21}\Delta_1+k_{22}\Delta_2+F_{2p}=0$

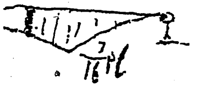
$$\begin{cases} 6i\Delta_1+\frac{3}{16}Pl=0 \\ \frac{11i}{l^2}\Delta_2-\frac{11}{16}P=0 \end{cases} \Rightarrow \begin{cases} \Delta_1=\frac{-Pl^2}{32i} \\ \Delta_2=+\frac{1}{16i}Pl^2 \end{cases}$$

$M=\bar{M}_1\Delta_1+\bar{M}_2\Delta_2+M_p$



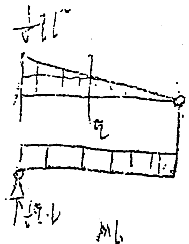
M图

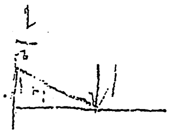




叠加时!

2003

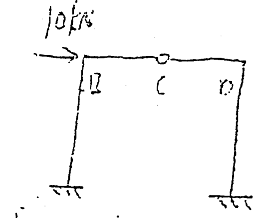


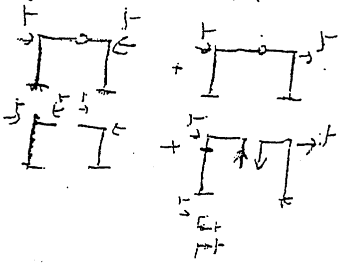


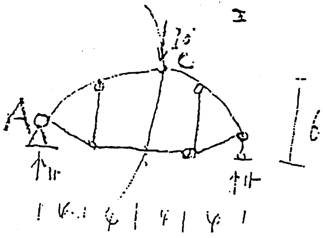
(X)

$\Delta=\frac{1}{EI}\left[\frac{1}{2}\times\frac{l}{2}\times\frac{l}{2}\times\left(\frac{ql^2}{8}+\frac{2}{3}\times\frac{ql^2}{8}\right)\right]$

7. [φ_C]







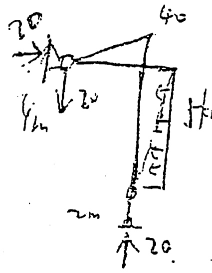
$\Sigma M_A=0$ $\quad$ $\Sigma M_C=0$

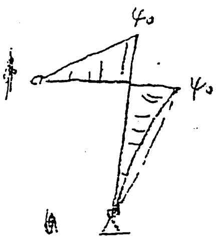
$N_{DE}\times2=\ldots$ $\quad$ $N_{DE}\times6=15\times8$

(X)

(X)

三、作附件和附属结构





1. [D] $\theta>1.0$ 无振动反射

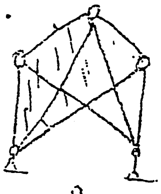
2. [D] $\frac{12EI}{l^3}=\frac{24EI}{l^3}$

[ ]

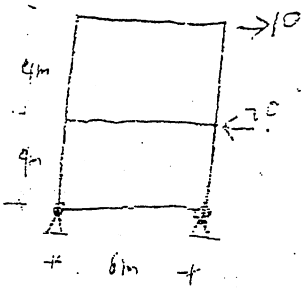
[B] $W=3\times8-2\times11-3=-1$

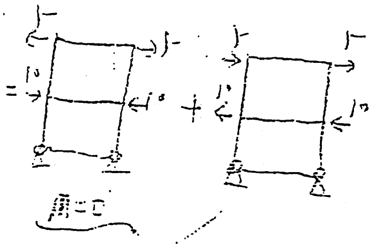
有多余约束

几何不变



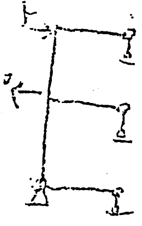
⑤ 消除——分解成对称和反对称 —

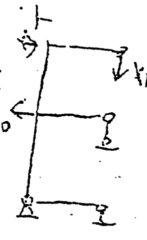


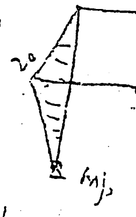


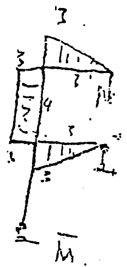
反对称结构

取半结构

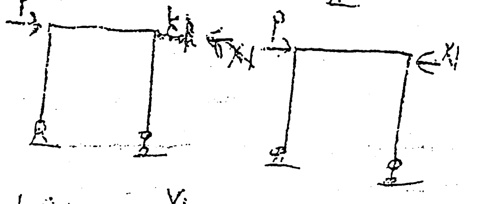








[C]

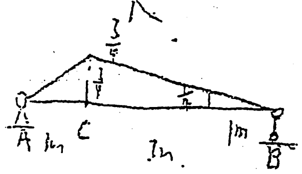


$\delta_{11}X_1+\Delta_{1P}=-\frac{X_1}{K}$

$X_1=\frac{-\Delta_{1P}}{\delta_{11}}=-\frac{\frac{1}{EI}\left(\frac{1}{2}\times4\times20\times3\right)}{\frac{1}{EI}\left(\frac{1}{3}\times3^2\times2+3\times3\times4\right)}=-\frac{20}{9}$

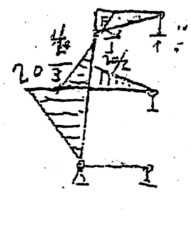
$M=\overline{M}_1X_1+M_P$

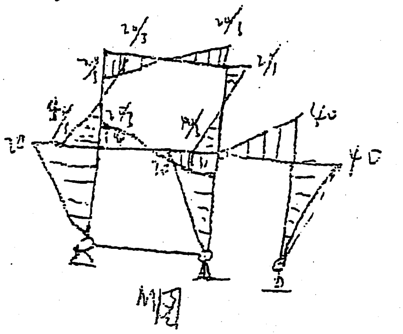
[B]



① $M=5\times\frac{3}{8}+28\times\frac{1}{4}=10.75$

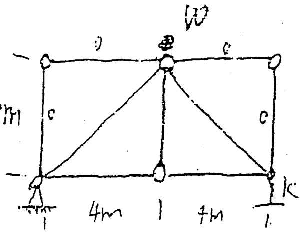
② $M=28\times\frac{3}{8}=21$





M图

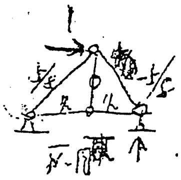
八题：



两个自由度.

$\delta_{11} = \sum \frac{\bar{F}^2}{EA} l - \sum \bar{R} C = \frac{1}{EA}\left(\frac{25}{36}\times5\times2 + \frac{4}{9}\times4\times2\right) + \frac{1}{2}\cdot\frac{1}{k}$

$= \frac{23}{3}\times10^{-6} = 7.7\times10^{-6}$



$\delta_{22} = \frac{1}{EA}\left(\frac{25}{64}\times5\times2 + \frac{1}{4}\times4\times2\right) + \frac{3}{8}\cdot\frac{3}{8}\cdot\frac{1}{k}$

$= 4.7\times10^{-6}$

$\delta_{12} = \delta_{21} = \sum \frac{\bar{F}_1\bar{F}_2}{EA} l - \sum \bar{R} C = \frac{1}{EA}\left(-\frac{5}{6}\cdot\frac{5}{8}\times5\times\ldots\right) + \frac{3}{8}\cdot\frac{1}{k} = 1.1\times10^{-6}$

设 $\lambda = \frac{1}{\omega^2}$

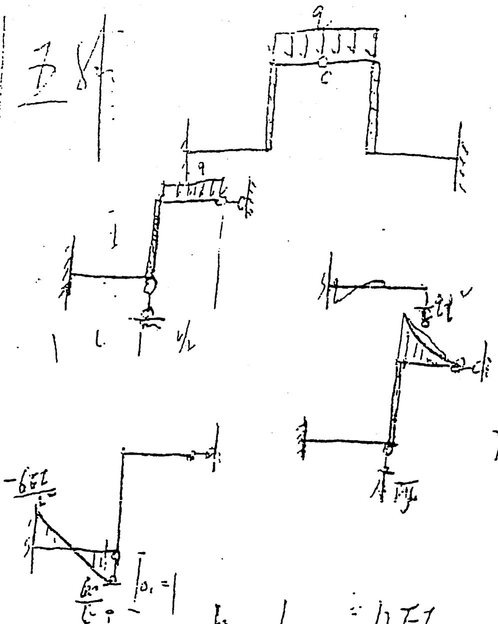
$\lambda_{1,2} = \frac{(m_1\delta_{11}+m_2\delta_{22}) \pm \sqrt{(m_1\delta_{11}+m_2\delta_{22})^2 - 4m_1m_2(\delta_{11}\delta_{22}-\delta_{12}\delta_{21})}}{2}$

$= \ldots$

$\omega_1 = \ldots$

$\omega_2 = \ldots$

五题

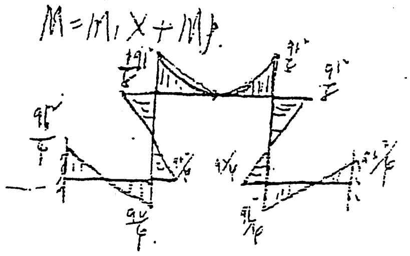


$F_P = -\frac{1}{2}ql$

$k_{11} = \frac{12EI}{l^3}$

$k_{11}\cdot\Delta_1 + F_P = 0 \Rightarrow \Delta_1 = \frac{ql^4}{24EI}$

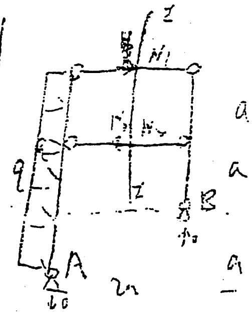
$M = \bar{M}_1 X + M_P$



M图

(可用力矩分配法！)

六题



由截断两杆之杆，①取右截面，由$\sum F_y=0 \Rightarrow V_B=0$.

由$\sum M_B=0$. $N_1\times2a+N_2\cdot a=0$ ①

②取左截面. 由$\sum M_B=0$.

$\frac{1}{2}q\cdot(3a)^2+N_1\times3a=N_2\times2a$ ②

联立①②

$\Rightarrow N_1=-\frac{9}{2}qa$ (拉力)

$N_2=9qa$ (压力).

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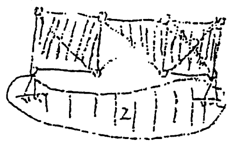
X) 2 (0) 3 (0)

$W = 3m - 2n - r = 3\times 11 - 2\times 17 - 4 = \ldots = -3$ 超静定

则 进行几何分析



M图不变 [B]

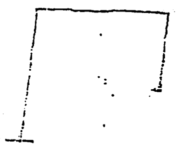
[B]

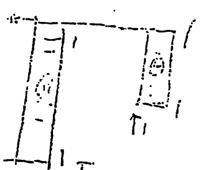
[A]

[C]

[C]

基本结构为





$\bar{N}$图 $\bar{M}$图

$t_0 = \dfrac{t_1+t_2}{2} = \dfrac{20-10}{2} = 5°C$

$\Delta t = t_1 - t_2 = 20-(-10) = 30°C$

$\Delta_{1t} = \alpha t_0 \int \bar{N} ds + \dfrac{\alpha \Delta t}{h} \int \bar{M} ds$

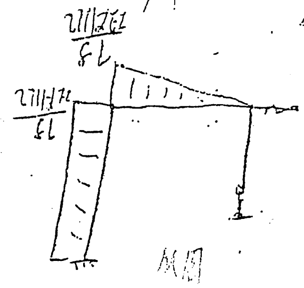
$= \alpha \cdot 5 \cdot (1\times l - \ldots) + \dfrac{\alpha \cdot 30}{l/10} \cdot \left(\dfrac{1}{2}\times l \times l + l \times l\right)$

$= \dfrac{5}{2}\alpha \cdot l + 5\times 90\alpha \cdot l = 452.5\alpha l$

$\delta_{11} = \dfrac{1}{EI}\cdot\left(\dfrac{1}{3}l^3 + l \times l \times l\right) = \dfrac{4l^3}{3EI}$

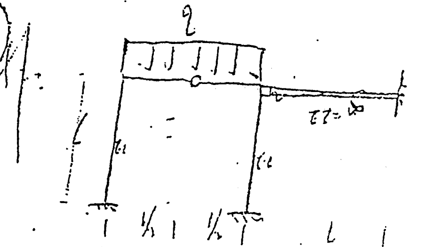
由 $\delta_{11} X_1 + \Delta_{1t} = 0$

$X_1 = \dfrac{-452.5\alpha l}{4l^3} \cdot 3EI = \dfrac{1357.5\alpha EI}{4l^2}$

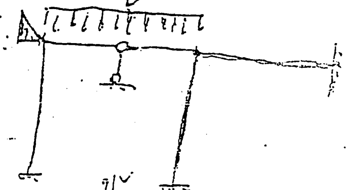


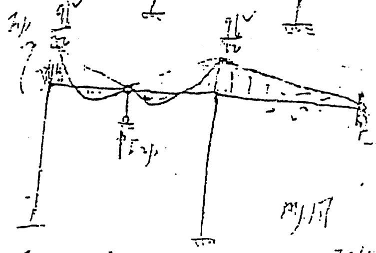
M图

(四)



基本体系

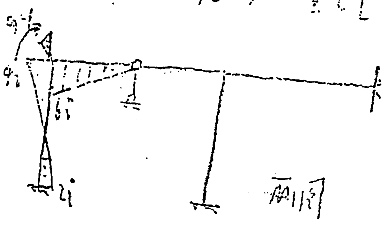




M_P图

$F_{1P} = -\dfrac{1}{8}q\left(\dfrac{l}{2}\right)^2 = -\dfrac{ql^2}{32}$

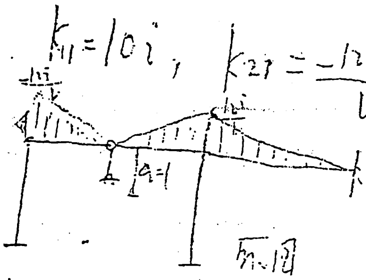
$F_{2P} = -\left(\dfrac{3}{16}ql + \dfrac{5}{16}ql\right) = -\dfrac{1}{2}ql$



$\bar{M}_1$图

设 $i = \dfrac{EI}{l}$

$k_{11} = 10i$, $k_{21} = -\dfrac{12i}{l}$



$\bar{M}_2$图

$k_{12} = -\dfrac{12i}{l}$ $k_{22} = \dfrac{48i}{l^2}$

$\begin{cases} k_{11}\Delta_1 + k_{12}\Delta_2 + F_{1P} = 0 \\ k_{21}\Delta_1 + k_{22}\Delta_2 + F_{2P} = 0 \end{cases}$ $\begin{cases}\Delta_1 = \\ \Delta_2 = \end{cases}$

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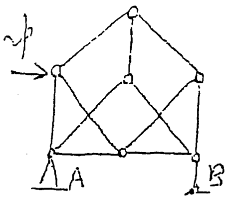
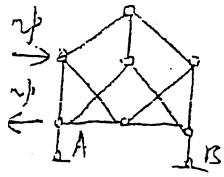
解：



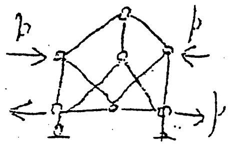
超静定桁架

由荷载及支座约束特征，可取为如下结构来求解



即将A处平支座去掉，并用相应之支座反力荷载P代之。

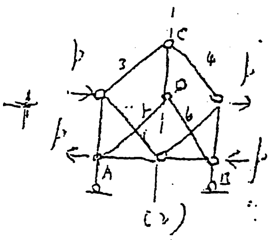
则结构化为对称结构，然后将荷载分解为



(1)

即对称结构受对称荷载

+

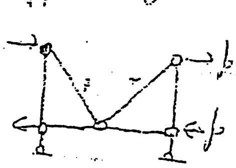


(2)

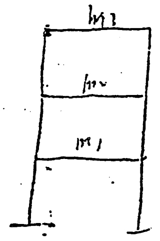
对称结构受反对称荷载.

(1)下，$N_1=N_2=0$.

(2)下，CD杆位于对称轴上，$N_{CD}=0$，并使相应的3、4、5、6杆均为0杆，故结构简化为



由结点平衡 $N_1=-\sqrt{2}P$(压) $N_2=\sqrt{2}P$(拉)



由多自由度体系的自由振动的主振型正交性，

第一阶主振型 $\{Y_l\}^T[M]\{Y_m\}=0$ $\quad l\neq m$, $l\in(1,3), m\in(1,3)$

设 $Y_3=\begin{pmatrix} a \\ b \\ 1 \end{pmatrix}$

故有 $(1/3,\ 2/3,\ 1)\begin{pmatrix} 270 & & \\ & 270 & \\ & & 180 \end{pmatrix}\begin{pmatrix} a \\ b \\ 1 \end{pmatrix}=0$

即 $a+2b+2=0$ ①

又有 $(-2/3,\ -2/3,\ 1)\begin{pmatrix} 270 & & \\ & 270 & \\ & & 180 \end{pmatrix}\begin{pmatrix} a \\ b \\ 1 \end{pmatrix}=0$

即 $a+b-1=0$ ②

由①② $\Rightarrow \begin{cases} a=4 \\ b=-3 \end{cases}$

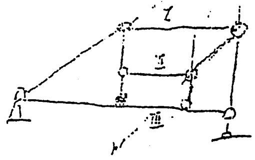
故第三振型 $\{Y_3\}=\begin{pmatrix} 4 \\ -3 \\ 1 \end{pmatrix}$

2001年入学试题解

一、

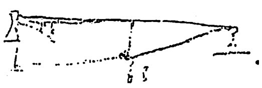
$m=9,\ n=12,\ C_0=3$

$W=27-24-3=0$



几何不变 [O]

二、(X)



[X]

[O]

[O]

三、1. [C]

$4\theta_B - \frac{6i}{l}\Delta_4 + 2i\theta_c = 0$

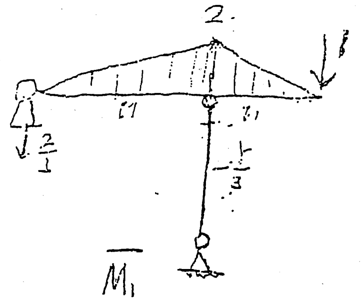
[D]

[D]

[C]

[B]

三、5. 略



$\overline{M_1}$

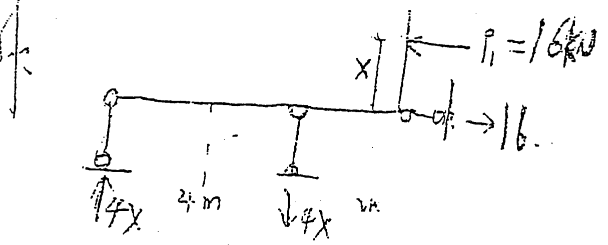
$\Delta_P = \delta_{11}\cdot P = \left(\int \frac{\overline{M_1}^2}{EI}ds + \frac{F^2 l}{EA}\right)P$

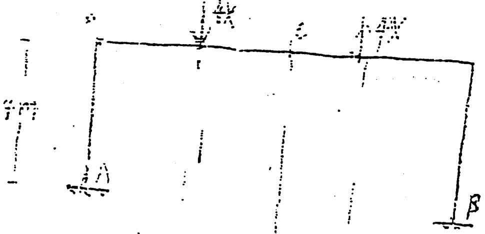
$= \left[\frac{1}{EI}\left(\frac{1}{2}\times 3\times 2\times\frac{2}{3}\times 2 + \frac{1}{2}\times 2\times 2\times\frac{2}{3}\times 2\right) + \frac{\left(\frac{5}{3}\right)^2\cdot 2}{EA}\right]P$

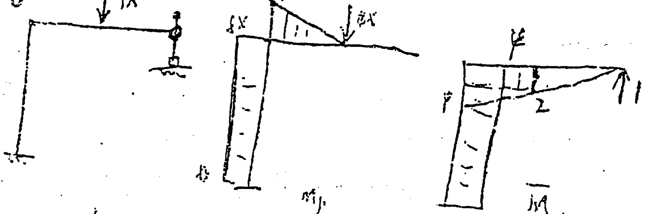
$=1$

$\Rightarrow P = 30.24\ kN$

四、8. 略







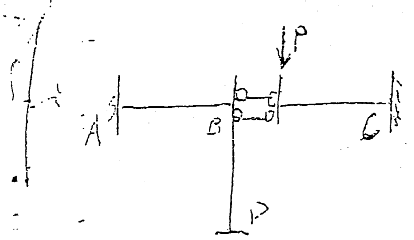
$\delta_{11} = \frac{1}{EI}\left(\frac{1}{2}\times 4\times 4\times\frac{2}{3}\times 4 + 4\times 4\times 4\right) = \frac{64\times 4}{3EI}$

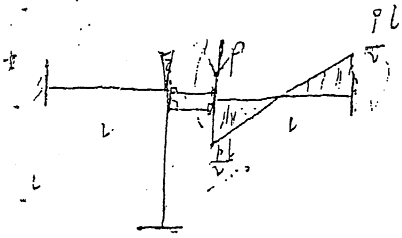
$\Delta_{1P} = -\frac{1}{EI}\left[4\times 4\times 8X + \frac{1}{2}\times 2\times 8X\times\left(2+\frac{2}{3}\cdot 2\right)\right] = -\frac{464X}{3EI}$

$X_1 = -\frac{\Delta_{1P}}{\delta_{11}} = \frac{464X}{64\times 4} = 1.8X$

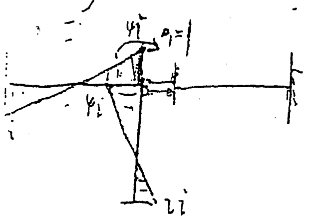
$M_b = 8X - 7.25X = 0.75X = 2.25\ kN\cdot m$

$X = 3m$





$F_{1P} = \frac{Pl}{2}$

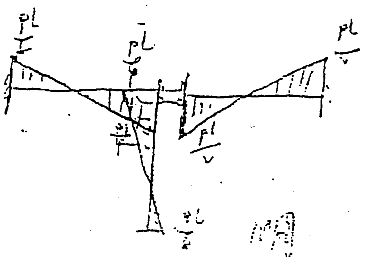


$i = \frac{EI}{l}$

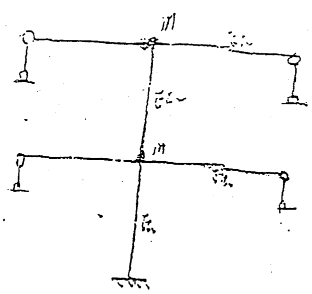
$k_{11} = 8i$

$k_{11}\Delta_1 + F_{1P} = 0$, $\Delta_1 = -\frac{Pl}{16i}$

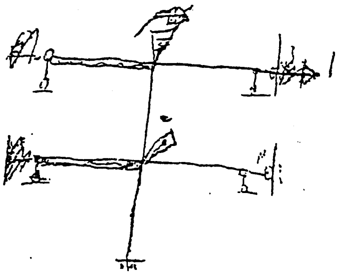
$M = \overline{M}_1 + M_P$



六

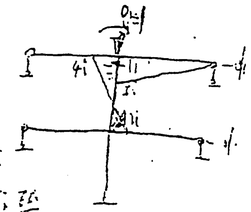


两个自由度



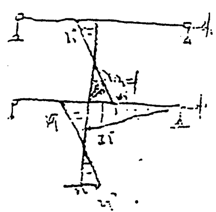
$F_{3P} = -1$

$F_{1P} = 0 = F_{2P} = F_{4P}$



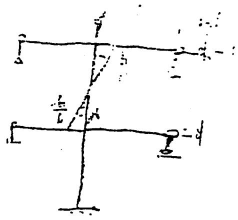
$k_{11} = 7i$, $k_{21} = 2i$

$k_{31} = -\frac{6i}{l}$, $k_{41} = \frac{6i}{l}$



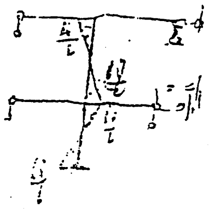
$k_{12} = 2i$, $k_{22} = 11i$

$k_{32} = -\frac{6i}{l}$ $k_{42} = 0$



$k_{13} = -\frac{6i}{l} = k_{31}$, $k_{23} = -\frac{6i}{l} = k_{32}$

$k_{33} = \frac{12i}{l^2}$ $k_{43} = \frac{-12i}{l^2}$



$k_{14} = \frac{6i}{l} = k_{41}$

$k_{24} = 0 = k_{42}$

$k_{34} = -\frac{12i}{l^2} = k_{43}$

$k_{44} = \frac{24i}{l^2}$

$\Delta_1 + k_{12}\Delta_2 + k_{13}\Delta_3 + k_{14}\Delta_4 + F_{1P} = 0$

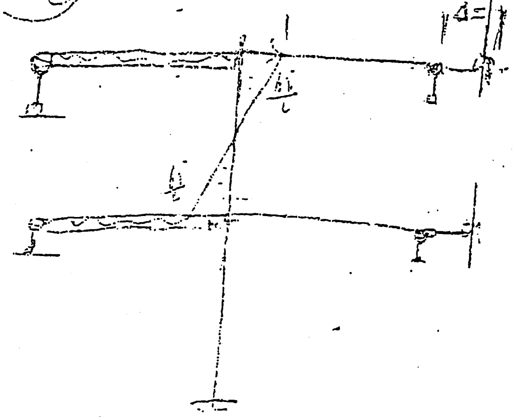
$\Delta_2 + k_{21}\Delta_1 + k_{23}\Delta_3 + k_{24}\Delta_4 + F_{2P} = 0$

$\Delta_3 + k_{31}\Delta_1 + k_{32}\Delta_2 + k_{34}\Delta_4 + F_{3P} = 0$

$\Delta_4 + k_{41}\Delta_1 + k_{42}\Delta_2 + k_{43}\Delta_3 + F_{4P} = 0$

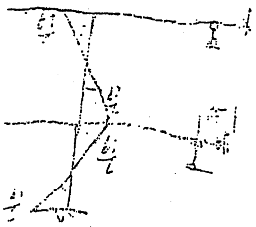
解联立 $\begin{cases} \Delta_1 = \\ \Delta_2 = \\ \Delta_3 = \\ \Delta_4 = \end{cases}$

(2) 改正



$F_{1P} = -1, \quad F_{2P} = 0$

$k_{11} = \frac{12i}{l^2} \quad k_{21} = -\frac{12i}{l^2}$

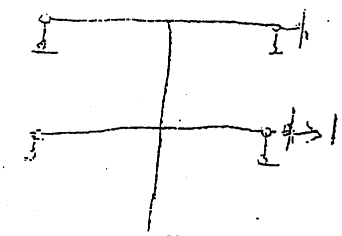


$k_{12} = -\frac{12i}{l^2}$

$k_{22} = \frac{24i}{l^2}$

$k_{11}\Delta_1 + k_{12}\Delta_2 + F_{1P} = 0$
$k_{12}\Delta_1 + k_{22}\Delta_2 + F_{2P} = 0$ ⇒ $\Delta_1 = \checkmark$, $\Delta_2 = \checkmark$

$M_1 = \bar{M}_1\Delta_1 + \bar{M}_2\Delta_2 + M$



$M_2 =$

固有频率 $\delta_{11}\ \delta_{12}\ \delta_{22}$ 解 $\lambda_1 = \ldots$ $= \omega_1 \ldots$

2006.

一、判断题

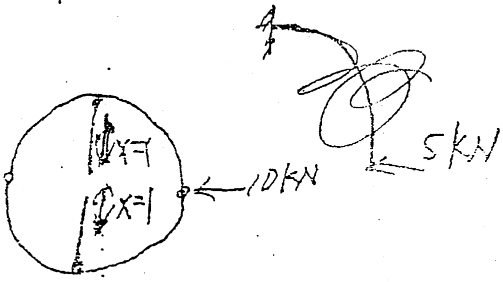
1. × 2. × 3. × 4. √ 5. √ 6. √

2. 填空

1. $\frac{3EI}{h}$ 2. 1.12 cm 3. 90 kN·m

4. $\sqrt{\frac{k_a + l k_b}{l m}}$

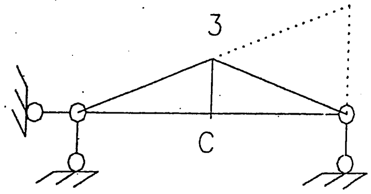
三、计算题



天津大学2008年结构力学真题参考答案

一、

1. 错误，反力互等定理反力系数数值相等，量纲也相同。
2. 错误，承受动力荷载的结构的位移不仅由结构刚度决定，还包括动力荷载的频率等因素。
3. 正确，绘出 M_C 影响线如图所示，$10kN$ 在 C 点时，$M_C=\dfrac{85}{2}kN\cdot m$；$5kN$ 在 C 点时，$M_C=25kN\cdot m$；$3kN$ 在 C 点时，$M_C=41kN\cdot m$；$7kN$ 在 C 点时，$M_C=24kN\cdot m$。

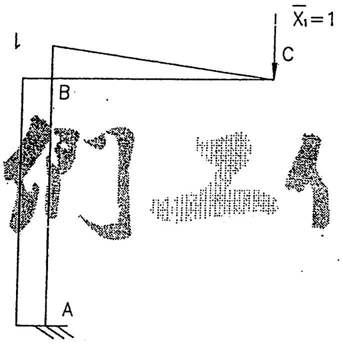


4. 正确。
5. 错误，在基本结构受竖向荷载时，轴向力为零，轴向多余未知力也就为零。

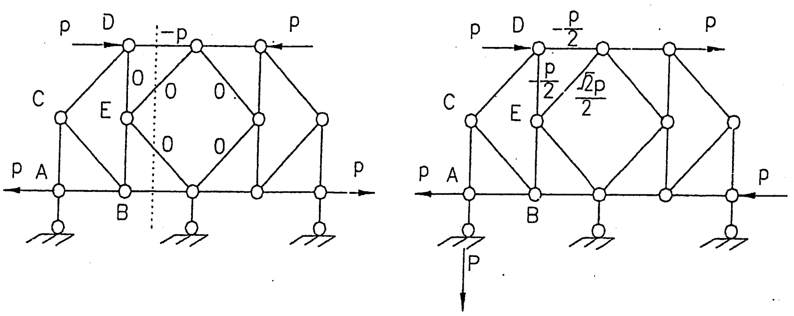
二、

1. 1；在阴影部分的角部加一个刚臂，各点位移就可以确定。
2. $61.85kN\cdot m$，$M=\overline{M_1}X_1$，$\overline{M_1}$ 不变；

$$\delta_{11}X_1+\Delta_{1t}=0,\delta_{11}=\frac{4l^3}{3EI},\Delta_{1t}=\frac{\alpha\Delta t}{h}\frac{3l^2}{2},X_1=-\frac{9EI\alpha\Delta t}{8hl}。$$



3. 几何不变体系，有一个多余约束，利用增加二元体的方法，HI 为多余约束。



4. $-\dfrac{P}{2}$

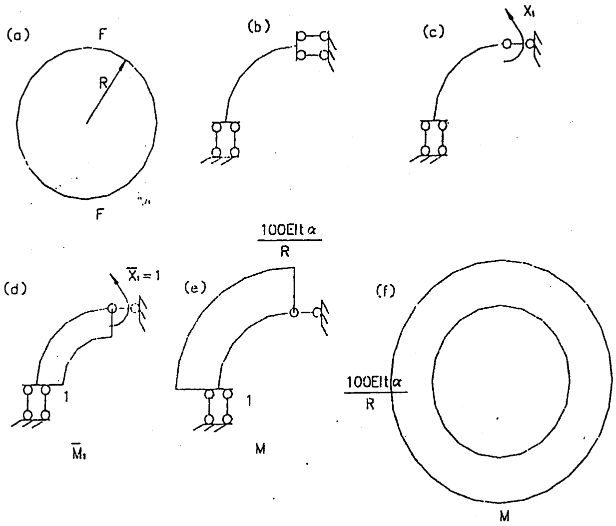
5. 从大变小，$\delta_{11}X_1+\Delta_{1P}=-\dfrac{1}{f}, X_1=-\dfrac{\dfrac{1}{f}+\Delta_{1P}}{\delta_{11}}, M=\overline{M_1}X_1+M_P$，$\overline{M_1}$ 和 M_P 不变。

三、

结构荷载有两个对称轴，取 $\dfrac{1}{4}$ 结构进行分析，典型方程 $\delta_{11}X_1+\Delta_{1t}=0$，

$$\delta_{11}=\frac{1}{EI}\int_0^{\frac{\pi}{2}}Rd\theta, \Delta_{1t}=\frac{\alpha\Delta t}{h}A_{\omega_{\overline{M}}}=\frac{10\alpha\Delta t}{R}\int_0^{\frac{\pi}{2}}Rd\theta=\frac{100\alpha}{R}\frac{\pi R}{2}$$

$$X_1=-\frac{100EI\alpha}{R}$$



利用 $M=\overline{M_1}X_1$ 绘出弯矩图，再利用对称性将整体结构的弯矩图绘出。

四、

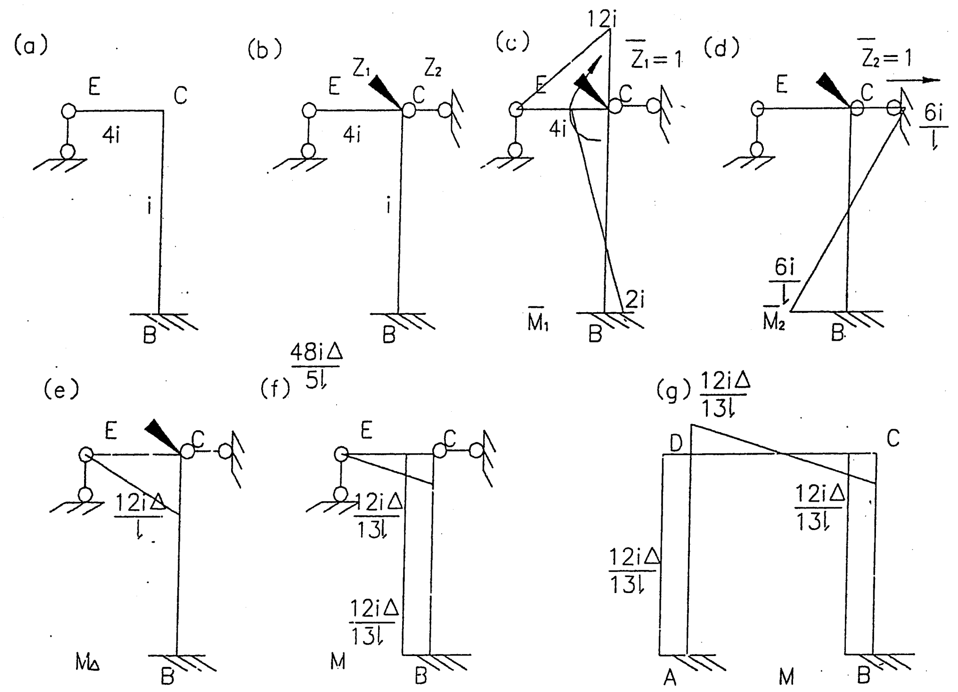
将位移荷载分为两组，一组为 AB 同时下移 $\dfrac{\Delta}{2}$，此时不产生弯矩，另一组为 A 上移 $\dfrac{\Delta}{2}$，B 下移 $\dfrac{\Delta}{2}$，

此时结构荷载反对称，取半结构。典型方程 $\left.\begin{aligned}r_{11}Z_1+r_{12}Z_2+R_{1\Delta}=0\\ r_{21}Z_1+r_{22}Z_2+R_{2\Delta}=0\end{aligned}\right\}$

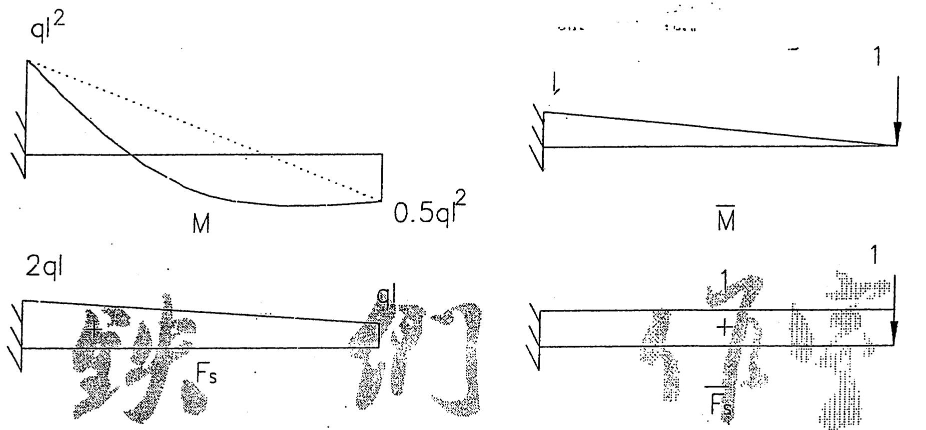
$$r_{11}=16i, r_{12}=r_{21}=-\frac{6i}{l}, r_{22}=\frac{12i}{l^2}, R_{1\Delta}=-\frac{12i\Delta}{l}, R_{2\Delta}=0$$

$$Z_1=\frac{12\Delta}{13l}, Z_2=\frac{6\Delta}{13}$$

由 $M=\overline{M_1}Z_1+\overline{M_2}Z_2+M_\Delta$ 绘出弯矩图，最后由结构荷载的对称性得出整体结构的弯矩图。



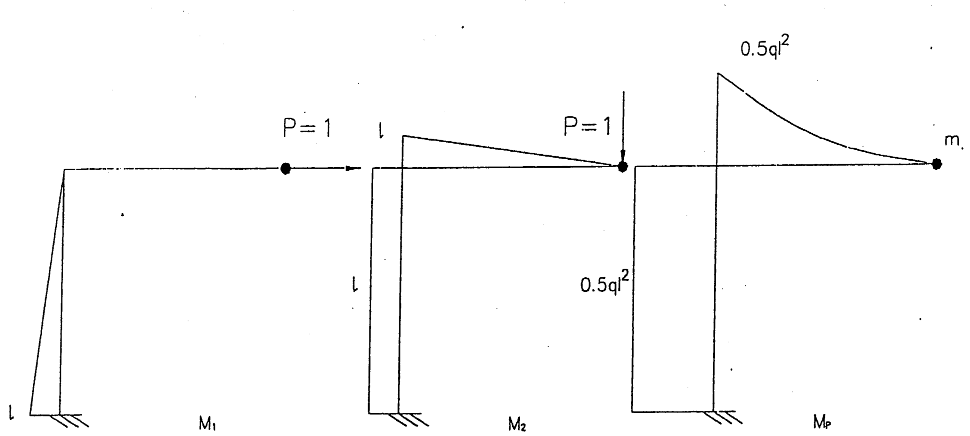
五、



$$\Delta_{KP}=\sum\int k\frac{\overline{Q_K}Q_P}{GA}ds+\sum\int\frac{\overline{M_K}M_P}{EI}ds=\frac{5ql^4}{24EI}+\frac{9ql^2}{5GA}$$

六、

该结构体系又两个自由度，水平方向和竖直方向



$$\delta_{11}=\frac{1}{EI}\frac{1}{2}l\times l\times\frac{2l}{3}=\frac{l^3}{3EI},\delta_{11}=\frac{1}{EI}\left(\frac{1}{2}l\times l\times\frac{2l}{3}+l\times l\times l\right)=\frac{4l^3}{3EI},\delta_{12}=\delta_{21}=\frac{1}{EI}\left(\frac{1}{2}l\times l\times l\right)=\frac{l^3}{2EI}$$

$$\Delta_{1P}=\frac{1}{EI}\frac{1}{2}l\times l\times\frac{ql^2}{2}=\frac{ql^4}{4EI},\Delta_{2P}=\frac{1}{EI}\left(l\times l\times\frac{ql^2}{2}+\frac{1}{3}\times l\times\frac{ql^2}{2}\times\frac{3l}{4}\right)=\frac{5ql^4}{8EI}$$

代入 $Y+\delta M\ddot{Y}=\Delta_P\sin\theta t$

运动方程为

$$\left.\begin{aligned}y_1+\frac{l^3}{3EI}m\ddot{y}_1+\frac{l^3}{2EI}m\ddot{y}_2=\frac{ql^4}{4EI}\sin\theta t\\ y_2+\frac{l^3}{2EI}m\ddot{y}_1+\frac{4l^3}{3EI}m\ddot{y}_2=\frac{5ql^4}{8EI}\sin\theta t\end{aligned}\right\}$$

天津大学 2009 年结构力学真题参考答案

一、

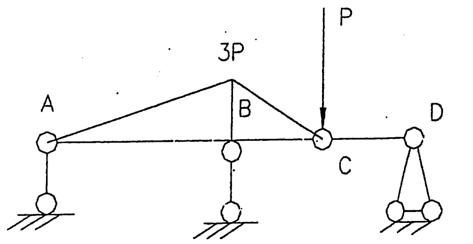
1. 错误，该体系为几何不变体系，且有一个多余约束。将 ABF、CDG 及基础各看成一个刚片，有三个铰联结，且链杆 AD 为多于约束。

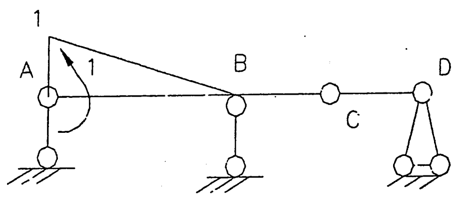
2. 错误，单位荷载法也可用于其它结构。

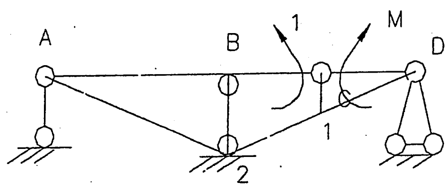
二、

1. 静定结构的计算用平衡条件计算即可，不涉及刚度。

2.





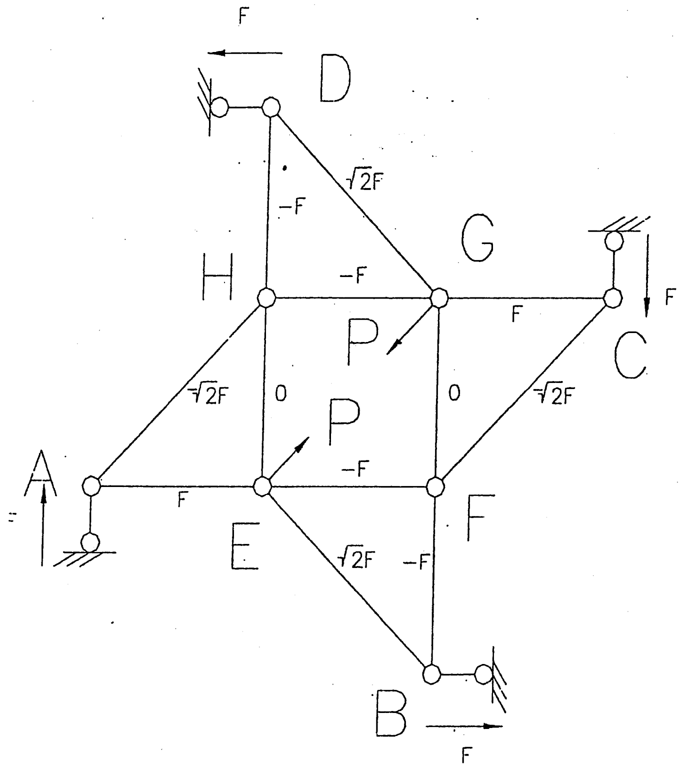


因 $\frac{1}{EI}\frac{1}{2}\times 6\times 3P\times\frac{1}{3}=0.001$，故 $-\frac{1}{EI}\frac{1}{2}\times 6\times 3P\times\frac{4}{3}-\frac{3}{6EI}\left(2\times 3P\times 2+3P\times 1\right)=-0.0065$（方向与图示相反）。

3. $\omega=\sqrt{\frac{4k}{11m}}$，该体系有一个自由度 θ，根据力矩平衡得 $11ma^2\ddot{\theta}+4ka^2\theta=0$，可得自振频率。

三、

利用水平方向和竖直方向的平衡，同时对中心取矩可得四个支座的力是大小相等的。



根据 G 点平衡可得 $F=\dfrac{\sqrt{2}}{2}P$，根据各点平衡得：

$$F=\frac{\sqrt{2}}{2}PF_{DG}=P,F_{HG}=-\frac{\sqrt{2}}{2}P,F_{HE}=0$$

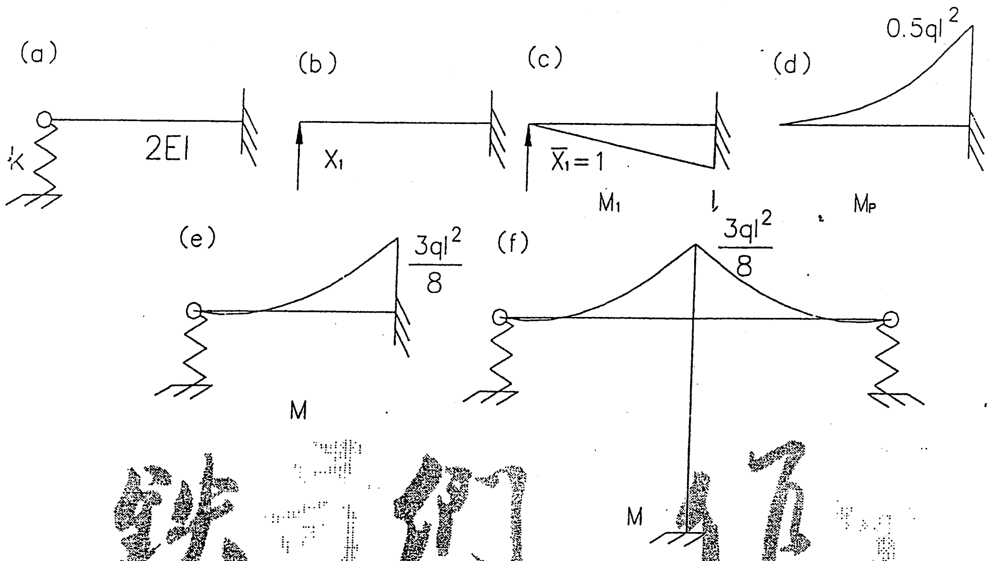
四、

结构荷载均对称，可取半结构进行计算，典型方程 $\delta_{11}X_1+\Delta_{1P}=-\dfrac{X_1}{k}$

$$\delta_{11}=\frac{1}{2EI}\times\frac{1}{2}\times l\times l\times\frac{2l}{3}=\frac{l^3}{6EI},\Delta_{1P}=-\frac{1}{2EI}\times\frac{1}{3}\times l\times\frac{1}{2}ql^2\times\frac{3l}{4}=-\frac{ql^4}{16EI}$$

得 $X_1=\dfrac{ql}{8}$

由 $M=\overline{M}_1X_1+M_P$ 绘出弯矩图，再由结构体系的对称性绘出整体结构的弯矩图。



五、

典型方程

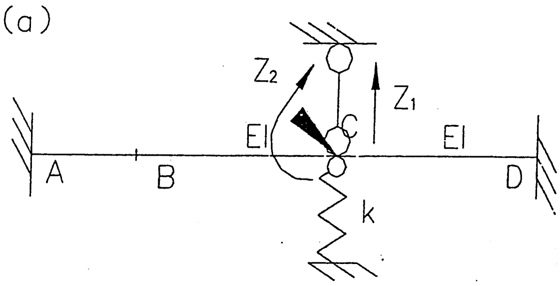
$$\left.\begin{aligned} r_{11}Z_1+r_{12}Z_2+R_{1\Delta}=0 \\ r_{21}Z_1+r_{22}Z_2+R_{2\Delta}=0 \end{aligned}\right\}$$

注意在 B 点既有角位移还有线位移，

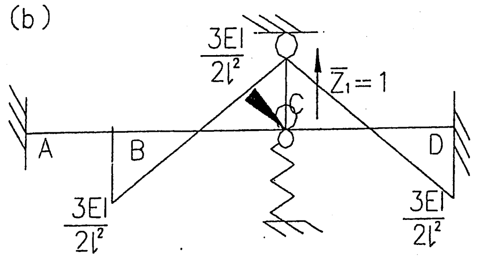
$$r_{11}=\frac{3EI}{l^3}+k=\frac{6EI}{l^3},r_{12}=r_{21}=0,r_{22}=\frac{4EI}{l},R_{1\Delta}=\frac{6EI\theta}{l},R_{2\Delta}=\frac{5EI\theta}{2l}$$

$$Z_1=-\frac{\theta l}{2},Z_2=-\frac{5\theta}{8}$$

利用 $M=\overline{M}_1Z_1+\overline{M}_2Z_2+M_\Delta$ 绘出结构的弯矩图，AB 段的弯矩由于 AC 段剪力未变，可直接由 BC 段弯矩图延伸过来。

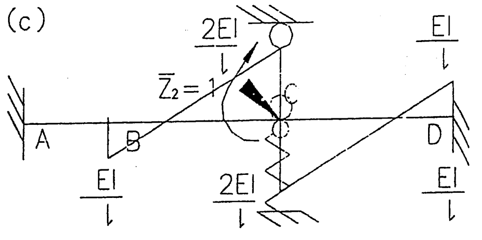


(b)



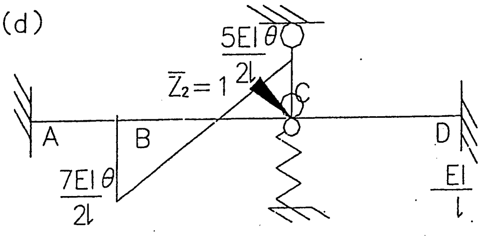
$\overline{M}_1$

(c)



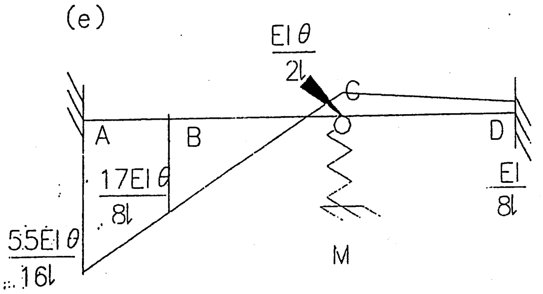
$\overline{M}_2$

(d)



M_Δ

(e)



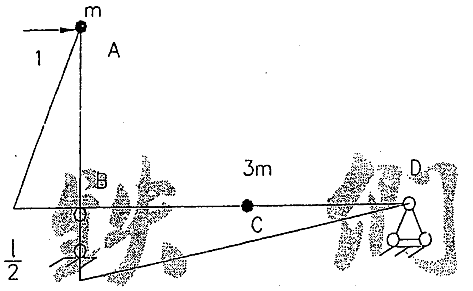
M

六、

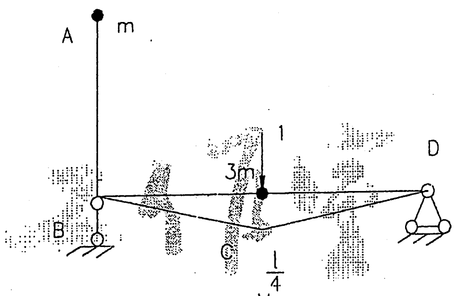
结构有两个自由度

$$\delta_{11}=\frac{1}{EI}\times\left(\frac{1}{2}\times\frac{l}{2}\times\frac{l}{2}\times\frac{l}{3}+\frac{1}{2}\times l\times\frac{l}{2}\times\frac{l}{3}\right)=\frac{l^3}{8EI},\delta_{22}=\frac{2}{EI}\times\frac{1}{2}\times\frac{l}{2}\times\frac{l}{4}\times\frac{l}{6}=\frac{l^3}{48EI}$$

$$\delta_{12}=\delta_{21}=\frac{1}{EI}\times\frac{1}{2}\times l\times\frac{l}{4}\times\frac{l}{4}=\frac{l^3}{32EI}$$



M_1



M_2

代入 $FM\ddot{Y}+Y=0$，$\left|FM-\frac{1}{\omega^2}I\right|=0$

设 $\lambda=\dfrac{ml^3}{32EI}$，$\begin{vmatrix}4\lambda-\dfrac{1}{\omega^2} & 3\\ 1 & 2\lambda-\dfrac{1}{\omega^2}\end{vmatrix}=0$ 得 $\omega_1=\sqrt{\dfrac{1}{5\lambda}}=\sqrt{\dfrac{32EI}{5ml^3}},\omega_1=\sqrt{\dfrac{1}{\lambda}}=\sqrt{\dfrac{32EI}{ml^3}}$

将 $\dfrac{1}{\omega^2}=\lambda,\dfrac{1}{\omega^2}=5\lambda$ 分别代入 $\left(FM-\dfrac{1}{\omega_i^2}I\right)\Phi_i=0$ 得

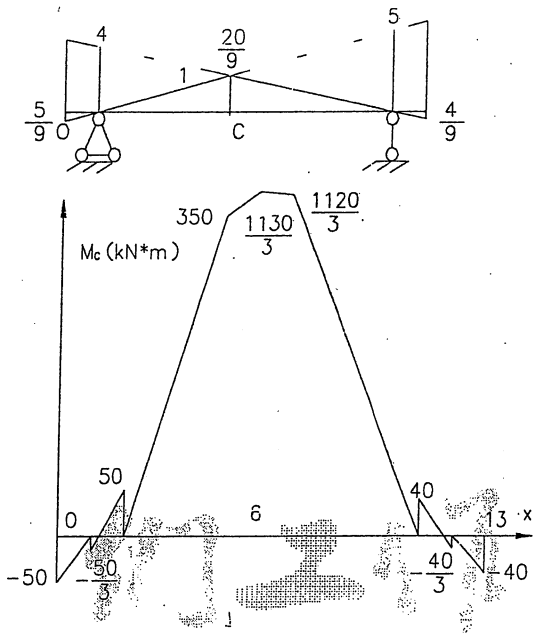
$$\begin{pmatrix}3\lambda & 3\\ 1 & \lambda\end{pmatrix}\begin{pmatrix}\varphi_1\\ 1\end{pmatrix}=0,\begin{pmatrix}-\lambda & 3\\ 1 & -3\lambda\end{pmatrix}\begin{pmatrix}\varphi_2\\ 1\end{pmatrix}=0$$

得 $\varphi_1=-1,\varphi_2=3$

振型 $\Phi_1=\begin{pmatrix}-1\\ 1\end{pmatrix},\Phi_2=\begin{pmatrix}3\\ 1\end{pmatrix}$。

七、

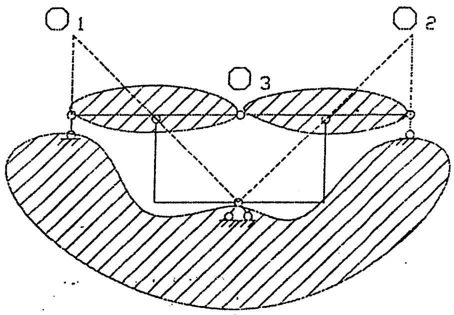
利用影响线做图，从荷载右端开始接触结构到左端离开结构，以右端荷载距离 0 点的距离为横轴，C 点的弯矩为纵轴，绘出 C 点弯矩有突变的点，然后用折线将点连接起来就为 C 点弯矩变化图，注意在涉及支座时，图形会有突变。



天津大学2010年结构力学真题参考答案

一、

1、几何不变体系且无多余约束。

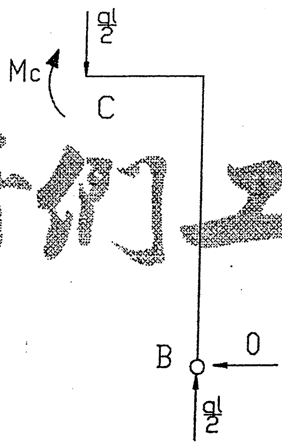


如图所示，根据三刚片规则，三铰不在一条直线上，可知该体系为几何不变体系且无多余约束。

2、$\dfrac{ql^2}{4}$。(↱ ↰)

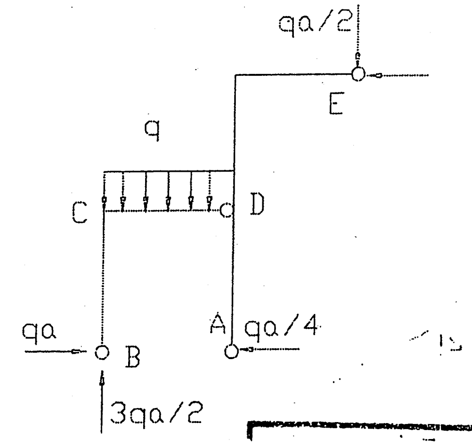
由三刚片规则知该结构为静定结构，且类似于三铰拱结构。分别取整体平衡和隔离体平衡即可得出结果。取整体平衡，对 A 点取矩得 B 支座竖向反力为 $ql/2$（↑）；取右隔离体平衡，对 B 点取矩得：

$$\frac{ql}{2}\cdot\frac{l}{2}=M_C$$



3、$\dfrac{qa}{4}$ (←)。

由对称性可知 F_{By} 为 $3qa/2$（↑）；取隔离体 BCD，对 D 点取矩得： $F_{Bx}=qa(\rightarrow)$；取隔离体 ABCDE，对 E 点取矩得：$F_A=qa/4$ (←)

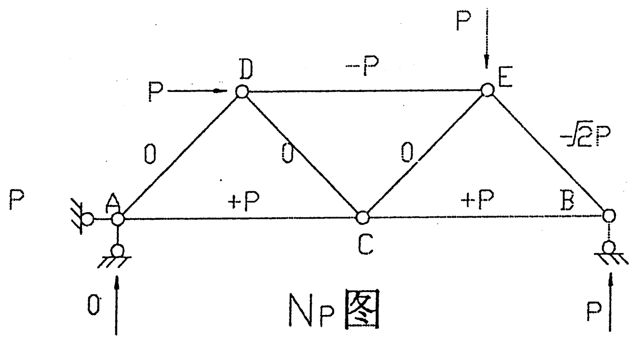


4、$\dfrac{(2+\sqrt{2})}{2EA}P$（顺时针）。

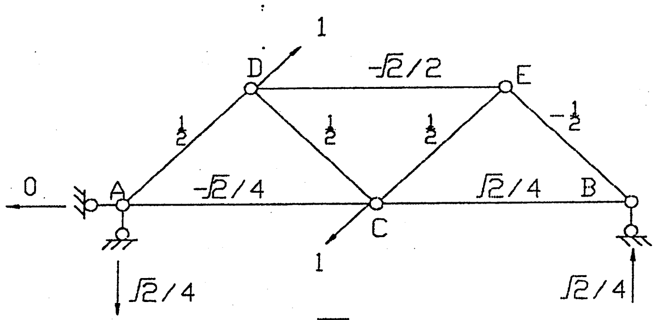
图乘得：$\theta=\dfrac{1}{EA}\left[(-P)(-\dfrac{\sqrt{2}}{2})\cdot 2a+(-\sqrt{2}P)(-\dfrac{1}{2})\cdot\sqrt{2}a\right]=\dfrac{(\sqrt{2}+1)Pa}{EA}$

相对转角为：$\dfrac{(\sqrt{2}+1)Pa}{EA}/\sqrt{2}a=\dfrac{(2+\sqrt{2})}{2EA}P$

注：或者在$\overline{N}$图中的杆两端施加一对大小为$\dfrac{1}{\sqrt{2}a}$、方向相反的集中力，得出的结果即为相对转角。



N_P图

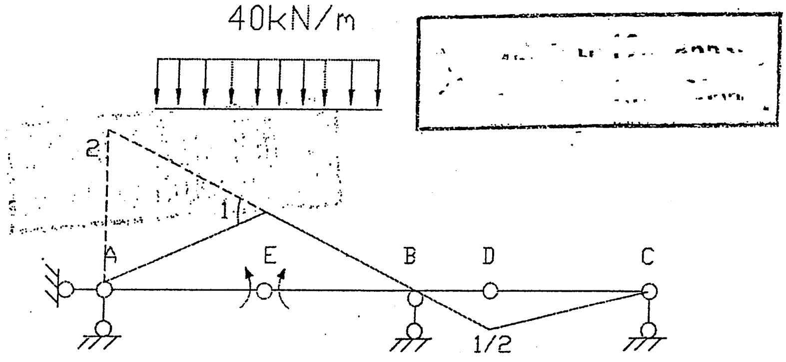


$\overline{N}$图

5、75 kN·m (下侧受拉)。

根据机动法作出 E 截面的弯矩影响线，如下图所示。

$$M_{E\max}=(\frac{1}{2}\cdot 4\cdot 1-\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{4}\cdot 2)\cdot 40=75\text{kN}\cdot\text{m}$$



6、$m\ddot{y}+\frac{3EI}{5l^3}y=\frac{3}{5}P(t)$

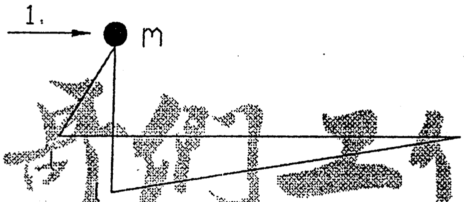
该题目为单自由度结构在简谐荷载作用下的强迫振动，且干扰力 $P(t)$ 不直接作用在质点上的情形。

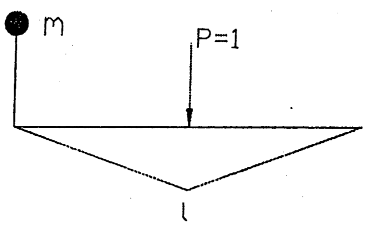
设任意时刻质点 m 处的位移为 y，则作用在质点 m 上的惯性力为 $F_I=-m\ddot{y}$，在惯性力 F_I 和干扰力 $P(t)$ 共同作用下，质点 m 处的位移为：

$$y=\delta_{11}F_I+\delta_{12}P(t)=\delta_{11}(-m\ddot{y})+\delta_{12}P(t)$$

将 $\delta_{11}=\frac{1}{EI}(\frac{1}{2}\cdot l\cdot l\cdot\frac{2l}{3}+\frac{1}{2}\cdot 4l\cdot l\cdot\frac{2l}{3})=\frac{5l^3}{3EI}$；$\delta_{12}=\frac{1}{EI}\frac{1}{2}\cdot 4l\cdot l\cdot\frac{l}{2}=\frac{l^3}{EI}$ 代入上式整理得：

$$m\ddot{y}+\frac{3EI}{5l^3}y=\frac{3}{5}P(t)$$





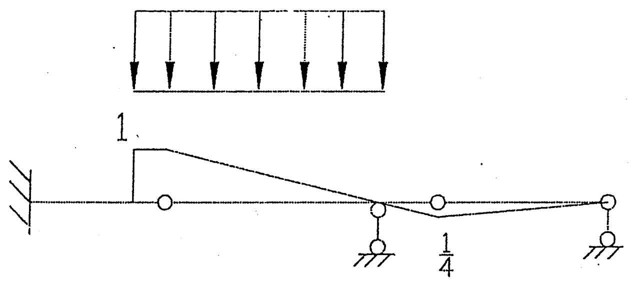
7、力法基本方程为：位移协调方程；或者 $\begin{matrix} \delta_{11}X_1+\delta_{12}X_2+\Delta_{1P}=0 \\ \delta_{21}X_1+\delta_{22}X_2+\Delta_{2P}=0 \end{matrix}$（以结构具有两个未知量为例）

位移法基本方程为：平衡方程；或者 $\begin{matrix} r_{11}Z_1+r_{12}Z_2+R_{1P}=0 \\ r_{21}Z_1+r_{22}Z_2+R_{2P}=0 \end{matrix}$（以结构具有两个未知量为例）

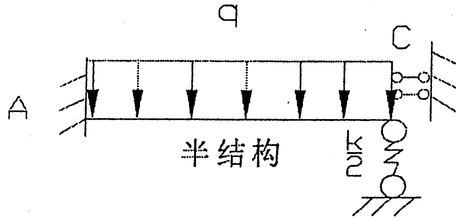
8、100kN。.

根据机动法，作出 K 截面剪力影响线，如下图所示。

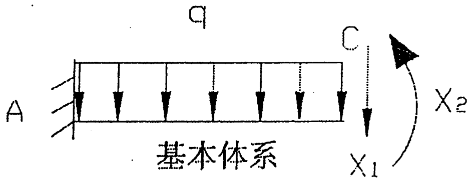
$$Q_{K\max}=(1\cdot 1+\frac{1}{2}\cdot 8\cdot 1)\times 20=100\text{kN}$$



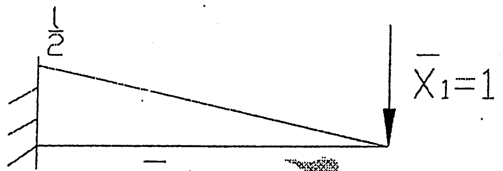
二、忽略轴向变形。半结构及基本体系如下图所示。



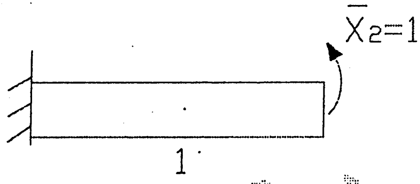
半结构



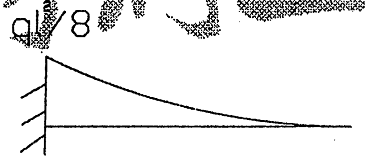
基本体系



$\overline{M}_1$图



$\overline{M}_2$图



M_P图

力法典型方程 $\left.\begin{matrix} \delta_{11}X_1+\delta_{12}X_2+\Delta_{1P}=-\dfrac{2X_1}{k} \\ \delta_{21}X_1+\delta_{22}X_2+\Delta_{2P}=0 \end{matrix}\right\}$

$$\delta_{11}=\frac{1}{EI}\frac{1}{2}\times\frac{l}{2}\times\frac{l}{2}\times\frac{l}{2}\times\frac{2}{3}=\frac{l^3}{24EI}\quad \delta_{22}=\frac{1}{EI}\cdot\frac{l}{2}\times1\times1=\frac{l}{2EI}$$

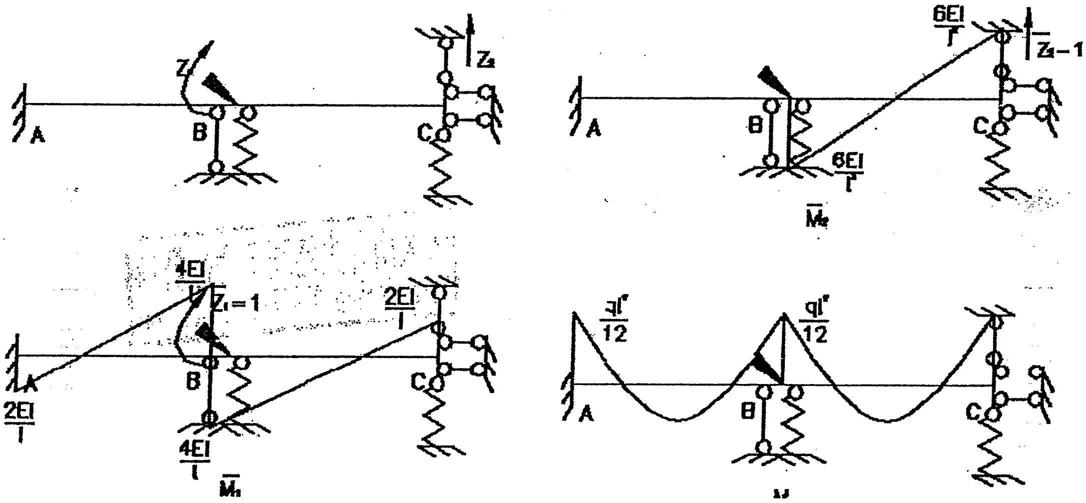
$$\delta_{12}=\delta_{21}=-\frac{1}{EI}\times\frac{1}{2}\times\frac{l}{2}\times\frac{l}{2}\times1=-\frac{l^2}{8EI}$$

$$\Delta_{1P}=\frac{1}{EI}\left(\frac{1}{3}\times\frac{l}{2}\times\frac{ql^2}{8}\times\frac{3}{4}\times\frac{l}{2}\right)=\frac{ql^4}{128EI},\ \Delta_{2P}=-\frac{1}{EI}\left(\frac{1}{3}\times\frac{l}{2}\times\frac{ql^2}{8}\times1\right)=-\frac{ql^3}{48EI}$$

由上图可知，AB梁中点C处截面弯矩为0，有$M_C=1\times X_2=0, X_2=0$

将$X_2=0$代入典型方程解得$k=\dfrac{384EI}{l^3}$。

三、

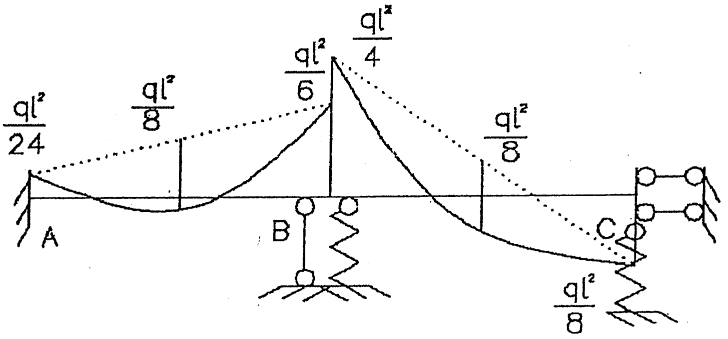


典型方程$\left.\begin{aligned}r_{11}Z_1+r_{12}Z_2+R_{1P}=0\\ r_{21}Z_1+r_{22}Z_2+R_{2P}=0\end{aligned}\right\}$

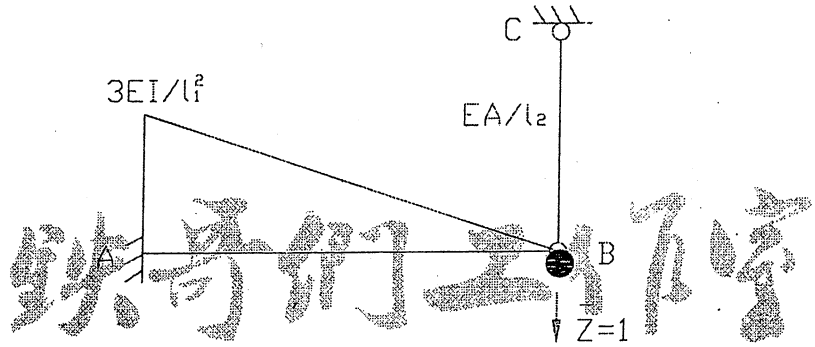
$$r_{11}=\frac{12EI}{l}, r_{12}=r_{21}=\frac{6EI}{l^2}, r_{22}=\frac{12EI}{l^3}+\frac{3EI}{l^3}=\frac{15EI}{l^3}, R_{1P}=0, R_{2P}=\frac{ql}{2}$$

$$Z_1=\frac{ql^3}{48EI}, Z_2=-\frac{ql^4}{24EI}$$

由$M=\overline{M_1}Z_1+\overline{M_2}Z_2+M_P$绘出弯矩图。



四、



（1）由刚度的物理意义得，

$$k=\frac{3EI}{l_1^3}+\frac{EA}{l_2}$$

$$k=51.25\times10^3 N/m$$

$$m=\frac{G}{g}=\frac{10\times10^3}{10}kg=10^3 kg$$

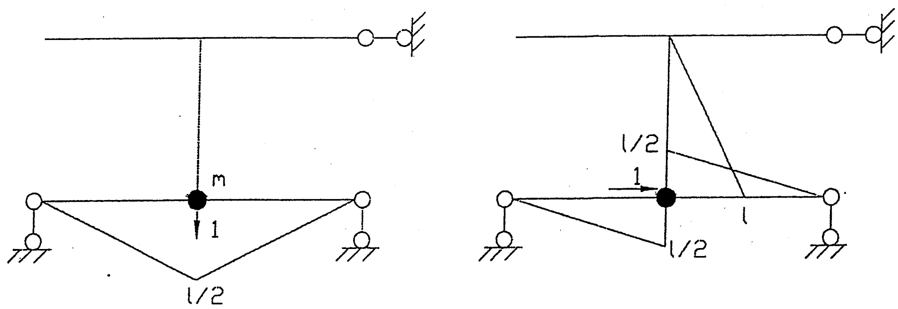
$$\omega=\sqrt{\frac{k}{m}}=7.2\frac{1}{s}$$

（2）结构为自由振动

$A=\sqrt{y_0^2+\left(\dot{y}/\omega\right)^2}=\sqrt{0.01^2+0.04^2/7.2^2}=0.011m$。

五、

结构具有两个自由度，利用柔度矩阵的物理意义求柔度矩阵。



$$\delta_{11}=\frac{1}{EI}2\times\frac{1}{2}\times l\times\frac{l}{2}\times\frac{l}{2}\times\frac{2}{3}=\frac{l^3}{6EI}\quad \delta_{22}=\frac{1}{EI}\left(\frac{1}{2}\times l\times l\times l\times\frac{2}{3}+\frac{1}{2}\times l\times\frac{l}{2}\times\frac{l}{2}\times\frac{2}{3}\times 2\right)=\frac{l^3}{2EI}$$

$$\delta_{12}=\delta_{21}=0$$

柔度矩阵 $F=\frac{l^3}{6EI}\begin{bmatrix}1 & 0\\ 0 & 3\end{bmatrix}$ 质量矩阵 $M=\begin{bmatrix}m & 0\\ 0 & m\end{bmatrix}$

$$FM\ddot{Y}+Y=0,\left|FM-\frac{1}{\omega^2}I\right|=0$$

$$\omega_1=\sqrt{\frac{2EI}{ml^3}},\omega_2=\sqrt{\frac{6EI}{ml^3}}$$